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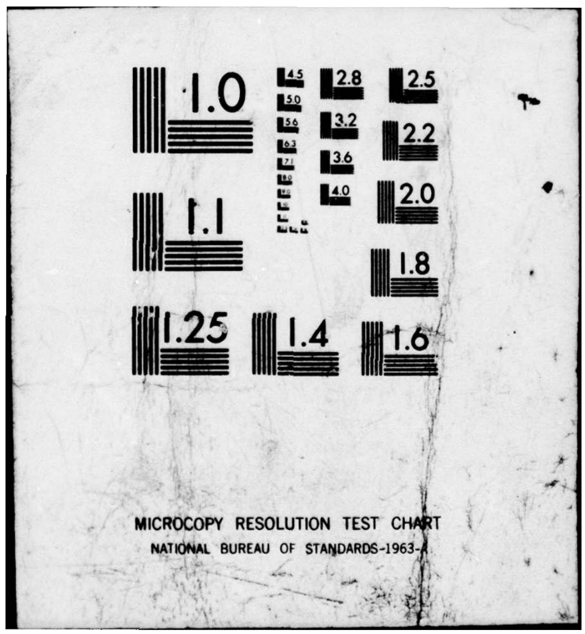
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AMBIENT NOISE EFFECTS IN THE MODELING OF DETECTION BY A FIELD OF SENSORS

Report to

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by

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ABSTRACT

This report describes a mathematical model for the ambient noise in the ocean caused by merchant ships. This kind of noise dominates in a frequency range in which many initial detections are made by modern sonar equipment and hence an understanding of it is of considerable interest.

A stochastic model for ambient noise is defined and its main properties are explored. The ambient noise process is a function both of time and of space, and it is shown to be a stationary stochastic process as a function of either of these variables. Its characteristic function is calculated as are its first two moments. It is shown that the ambient noise process as a function of time has approximately an exponential autocovariance function and it may be satisfactorily modeled as a Gauss-Markov process. Further, it is shown that the correlation between the ambient noise at two points depends to a good approximation only on the propagation loss function and on the distance between these two points (and not materially on the density or loudness of the noise field). Finally, an effective and practical method for modeling the multidimensional detection process is presented.

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PREFACE

This is a report by Daniel H. Wagner, Associates to the Naval Analysis Division of the Office of Naval Research on a research investigation under ONR Contract No. N00014-76-C-0637. The report is addressed to the modeling of the sonar detection process when there are multiple sensors, such as a field of sonobuoys. The approach taken is to assume that the random component of signal excess has three parts: an independent component, a correlated component, and a residual component.

It is hypothesized that the correlated component is principally due to ambient noise, and a mathematical model for ambient noise due to shipping is defined and its main properties discussed. A convenient method for simulating sample paths of the correlated ambient noise fluctuations is then given.

We would like to acknowledge the cooperation and support we have received during this work from Mr. James G. Smith and Mr. J. Randolph Simpson of the Naval Analysis Division of the Office of Naval Research.

SUMMARY

In sonar detection modeling it is assumed that a detection of a target occurs if

$$x(t) + \xi(t) > 0$$

at some time t, where $x(t) = FOM(t) - N_W(R(t))$ is the deterministic component of signal excess at the sensor at time t when the range between sensor and target is R(t), and $\xi(t)$ is its random component. The term FOM(t) represents the sonar figure-of-merit at time t, and N_W is the propagation loss function. Various models for the random component are in use; these and the available results in computing cumulative detection probability (cdp) and mean time to detect are given in reference [a].

Now suppose there are n sensors, and n corresponding signal excess histories $x_1, \ldots, x_n$. The problem is to hypothesize a structure for the corresponding random components $\xi_1, \ldots, \xi_n$. There are two easy ways out:

(1) the ξ_i's are all independent;

(2) the ξ_i's are completely dependent, i.e., $\xi_1 = \xi_2 = \ldots = \xi_n$.

Another possibility is to compute some quantity of interest such as cdp or mean time to detect under assumptions (1) and (2) and average the results (weighting them equally, as in reference [b], or almost equally, as in reference [c]). Of course, none of these is correct; the right amount of correlation is surely range dependent, as we shall see.

We will hypothesize that each ξ_i is of the form

$$\xi_i(t) = \xi_c(t) + \eta_i(t) + \eta_i'(t)$$

where ξ_c is identical at each sensor, η_i' is independent of η_k' for all $i \neq k$ and of η_k for all k, η_i' is independent of ξ_c for all i, η_i is independent of ξ_c for all i,

but $\eta_i(t)$ and $\eta_k(s)$ are correlated. For these reasons ξ_c may be called the residual component, η_i the correlated component, and η_i' the independent component. The process ξ_c represents fluctuations due to the target, η_i represents fluctuations due to ambient noise, and η_i' represents fluctuations due to sensor performance and propagation.

In Chapter II a mathematical model for ambient noise due to merchant shipping is given. It is assumed that the ambient noise at a point P at time t is given by

$$\eta(P, t) = 10 \log \Sigma_j Z_j f(R_j(P, t))$$

where Z_j is the power emitted by the j^{th} noise source, f is the power loss function and $R_j(P, t)$ is the distance between the j^{th} noise source and the point P at time t. The identification $\eta_i(t) = \eta(P_i, t) - E\,\eta(P_i, t)$, where P_i is the position of the i^{th} sensor, will be made.

The characteristic function of the related process $\zeta = 10^{\eta/10}$ is computed as are its mean and variance. It is shown that the inclusion of the effects of attenuation in the propagation loss function may be crucial.

In Chapter III it is shown that η is a stationary stochastic process as a function of time. It is also argued that η as a function of t is approximately a Gauss-Markov process, and a parametric estimate for the relaxation time is given.

In Chapter IV the correlation between $\eta(P, t)$ and $\eta(Q, t)$ is shown to be approximately independent of everything but the propagation loss function and the distance between P and Q. A method for estimating $\mathscr{C}(u)$, the correlation in ambient noise between two points separated by distance u, is described and applied to a set of eight environments. The qualitative effects of convergence zones on the function $\mathscr{C}$ are described.

Finally, in Chapter V, a method for simulating the correlated processes $\eta_1, \ldots, \eta_n$ is given. This involves simulating n independent copies of the η process and performing a matrix transformation which will result in n processes with the desired correlation structure. The matrix needed for this transformation turns out to be the square root of the matrix $((\mathscr{C}(r_{ik})))$, where r_{ik} is the distance between sensor i and sensor k.

Our model for ambient noise due to shipping assumes a homogeneous density of noise sources. That is, we do not consider cases where there are significant departures from homogeneity such as heavily traveled shipping lanes. Most of our results will no doubt have to be modified in order to treat non-homogeneous noise fields, but these extensions will be postponed for future consideration.

THE EFFECTS OF AMBIENT NOISE ON THE DETECTION PROCESS AND THE MODELING OF DETECTION BY A FIELD OF SENSORS

CHAPTER I

INTRODUCTION

In a sonar detection model one hypothesizes that a sensor is in contact with a target at time t if

$$x(t) + \xi(t) \geq 0$$

where x is a deterministic function (called the signal excess) obtained from the sonar equation

$$x(t) = L_S(t) - \left(L_N(t) - N_{DI}(t)\right) - N_{RD}(t) - N_W\left(R(t)\right)$$

$$= \mathrm{FOM}(t) - N_W\left(R(t)\right)$$

and ξ is a stationary stochastic process representing the combined uncertainties and fluctuations in the components of the sonar equation. In the above, FOM is called the figure of merit, L_S is radiated noise, L_N is ambient or self noise, N_{DI} is the directivity index, N_{RD} is recognition differential, R is a function giving distance between target and sensor, and N_W is the propagation loss function. The sonar equation and its components are discussed fully in texts such as reference [d].

Given a specific mathematical model for ξ one can then in theory compute such items of interest as cumulative detection probability,

$$\mathrm{cdp}(t) = \Pr\{x(s) + \xi(s) \geq 0 \text{ for some } s \leq t\}, \tag{I-1}$$

mean time to detect, mean time in contact, etc. See references [a] or [f] for surveys of results available for this single-sensor model, and reference [e] for a discussion of the need for methods of estimating cdp.

In this report we discuss a mathematical model for that component of ξ which is due to fluctuations in ambient noise due to shipping. Thus we address fluctuations contributed by the function L_N, and then only when L_N is dominated by contributions from shipping sources. This is generally in the 50-500 Hz frequency band, an interval of some interest in sonar detection. Fluctuations of the other components may be modeled by the methods of reference [a] until such time as mathematical models for them are available.

This ambient noise model is applied to derive a method for treating the problem of computing cdp for a field of sensors. That is, suppose there are n sensors and a target's behavior produces n signal excess histories $x_1, \ldots, x_n$ and n corresponding fluctuation process sample paths $\xi_1, \ldots, \xi_n$. Then we are often interested in computing the function

$$\mathrm{cdp}(t) = \Pr\{x_i(s) + \xi_i(s) \geq 0 \text{ for some } s \leq t \text{ and some } i = 1, \ldots, n\},$$

which is the cumulative probability of detection by <u>some</u> member of the field of n sensors. (From here on cdp has this more general definition, as opposed to (I-1).)

The problem in computing cdp is that some assumption has to be made about the joint probability distribution of the ξ_i. For example, if ξ_i is independent* of ξ_k for all $i \neq k$, then

$$\mathrm{cdp}(t) = 1 - \prod_{i=1}^{n} \left(1 - \mathrm{cdp}_i(t)\right),$$

where

$$\mathrm{cdp}_i(t) = \Pr\{x_i(s) + \xi_i(s) \geq 0 \text{ for some } s \leq t\}.$$

On the other hand if the ξ_i are completely dependent so that $\xi_1 = \xi_2 = \ldots = \xi_n$, then

$$\mathrm{cdp}(t) = \Pr\{\max_{1 \leq i \leq n} x_i(s) + \xi_1(s) \geq 0 \text{ for some } s \leq t\}$$

* The process ξ_i is independent of ξ_k if the random variables $\xi_i(s)$ and $\xi_k(t)$ are independent for all s and t.

which is of the form (I-1) with $x(s) = \max_{1 \leq i \leq n} x_i(s)$.

In reality the correlation between $\xi_i(t)$ and $\xi_k(t)$ for a given t is somewhere between 0 and 1. That is because each sensor is subject to a variety of types of fluctuation some of which are shared to varying degrees by the other sensors and some of which are peculiar to the sensor itself. We hypothesize that each ξ_i is of the form

$$\xi_i(t) = \xi_c(t) + \eta_i(t) + \eta_i'(t)$$

where ξ_c is identical at each sensor, η_i' is independent of η_k' for all $i \neq k$ and of η_k for all k, η_i' is independent of ξ_c for all i, η_i is independent of ξ_c for all i, but $\eta_i(t)$ and $\eta_k(s)$ are correlated. For these reasons ξ_c may be called the residual component, η_i the correlated component, and η_i' the independent component. We envision ξ_c as deriving from fluctuations due to the target, η_i' as deriving primarily from fluctuations in N_{DI}, N_{RD}, and propagation loss anomalies in the immediate vicinity of the sensor, and η_i as deriving primarily from fluctuations in the ambient noise field. As such η_i and η_k will be positively correlated since they share the same ambient noise field, albeit a different view of it.

Now suppose through theory or experiment one has an estimate of ρ_{ik}, the correlation between $\eta_i(t)$ and $\eta_k(t)$ for each $t \geq 0$. How does one go from the matrix $((\rho_{ik}))$ *of correlations* to a method for simulating sample paths $\eta_1, \ldots, \eta_n$? This question is resolved in the final chapter. In the intervening chapters we define the ambient noise process, investigate its elementary properties, show that it is a stationary stochastic process and that it can be adequately approximated as a Gauss-Markov process.

It should be mentioned here that methods for computing cdp analytically are <u>not</u> available except for the cases where the ξ_i are all independent or completely dependent. For the more realistic case of correlated ξ_i's, Monte Carlo methods must be used.

CHAPTER II

THE AMBIENT NOISE MODEL

It ceased; yet still the sails made on
A pleasant noise till noon,
A noise like of a hidden brook
In the leafy month of June,
That to the sleeping woods all night
Singeth a quiet tune.

Samuel Taylor Coleridge,
The Rime of the Ancient Mariner

Suppose that at time t, ambient noise sources to be thought of as ships are located at points $\{S_j(t)\}_{j\in J}$ in the Euclidean plane where J is a countable index set. The j^{th} ship is assumed to be an omnidirectional noise source producing a noise level of N_j db (referenced to some fixed intensity, at some fixed frequency of interest). The j^{th} ship is assumed to be on a fixed course θ_j at a constant speed v_j.

Then for each point in the plane and each time t there is defined a random variable

$$\eta(P,t) = 10 \log \sum_{j\in J} 10^{\left(N_j - N_W\left(d(P,S_j(t))\right)\right)/10}$$

where d(P, Q) is the distance between the points P and Q. (In all that follows log will denote common logarithms and ln will denote natural logarithms.) The quantity $\eta(P,t)$ is the magnitude, in db, of noise reaching the point P at time t, and η is a stochastic process as a function of P and t. It will sometimes be referred to as $\eta(P)$ when t is fixed, or as $\eta(t)$ when P is fixed. To relate it to the previous chapter, $\eta_i(t)$ will be defined as $\eta(P_i,t) - E\,\eta(P_i,t)$ where P_i is the location of the i^{th} sensor.

We will also have need of the power process ζ defined by

$$\zeta(P,t) = \sum_{j\in J} 10^{\left(N_j - N_W\left(d\left(P, S_j(t)\right)\right)\right)/10}$$

$$= 10^{\eta(P,t)/10}.$$

The initial positions $S_j(0)$ of the noise sources at time $t=0$ will be assumed chosen in one of two ways. For the Poisson noise process we assume that $\{S_j(0)\}_{j\in J}$ is a sample from a 2-dimensional Poisson process with intensity $\lambda > 0$. For the bounded noise process we assume that some fixed finite number $M > 0$ of points $\{S_j(0)\}_{j=1}^{M}$ are selected independently and according to the uniform distribution on some bounded measurable subset $\mathscr{B}$ of the plane.

The initial courses θ_j will be assumed drawn independently from the uniform distribution on $[0, 2\pi)$. These courses remain constant for a Poisson noise process. For a bounded noise process the noise sources are assumed to reflect off the boundaries of $\mathscr{B}$ when they are run into. The speeds v_j and noise levels N_j are assumed drawn independently from distributions F_V and F_N, respectively. Both v_j and N_j remain constant for all $t \geq 0$.

Before proceeding to describe the probability distribution of $\eta(P,t)$ for fixed P and t, we need to mention some properties of the spatial Poisson process. Suppose $\{S_j\}_{j\in J}$ is a sample from a spatial Poisson process with parameter λ, the mean number of points per unit area. If A is a measurable subset of the plane let $\mu(A)$ denote its area and let $\chi(A)$ be the number of points from the set $\{S_j\}_{j\in J}$ that are in A. Then the following properties will be useful:

(i) The random variable $\chi(A)$ has a Poisson distribution with parameter $\lambda\mu(A)$, that is,

$$\Pr\{\chi(A) = k\} = e^{-\lambda\mu(A)} \frac{(\lambda\mu(A))^k}{k!}, \quad k = 0, 1, \ldots .$$

(ii) If A_1 and A_2 are disjoint, then $\chi(A_1)$ and $\chi(A_2)$ are independent random variables.

(iii) Given that $\chi(A) = k$, then the k points in A are distributed independently and according to a uniform distribution on A.

To derive the characteristic function φ of $\zeta(P,t)$ we note that P can be taken as the origin, without loss of generality, and that ζ is of the form

$$\zeta = \sum_{j\in J} Z_j f(r_j)$$

where

$$Z_j = 10^{N_j/10},$$

r_j is the distance between $S_j(t)$ and the origin, and

$$f(r) = 10^{-N_W(r)/10}$$

is called the power loss factor at range r.

Theorem. Let $\{Z_j\}_{j \in J}$ be a sequence of independent non-negative random variables with common distribution function F_Z, let $\{S_j\}_{j \in J}$ be a sample from a spatial Poisson process with parameter λ and let r_j be the distance of S_j from the origin, for $j \in J$. Then

$$\zeta = \sum_{j \in J} Z_j \, f(r_j)$$

has the characteristic function φ given by

$$\varphi(y) = \exp\{2\pi\lambda \int_0^\infty \int_0^\infty (e^{izyf(r)} - 1)\, r\, dr\, d\, F_Z(z)\}. \tag{II-1}$$

Proof. Let ζ_m denote the random variable

$$\zeta_m = \sum_{r_j < m} Z_j f(r_j)$$

with characteristic function φ_m, for $m = 1, 2, \ldots$. Then $\{\zeta_m\}$ converges a. s. to ζ so that by the dominated convergence theorem $\varphi_m(y)$ converges to $\varphi(y)$ for all y. Let B_m denote the disc with radius m, centered at the origin. Then

$$\varphi_m(y) = E\, e^{iy\zeta_m}$$

$$= \sum_{k=0}^{\infty} E\left(e^{iy\zeta_m} \mid \chi(B_m) = k\right) \Pr\{\chi(B_m) = k\}$$

$$= \sum_{k=0}^{\infty} E\left(e^{iy\zeta_m} \mid \chi(B_m) = 1\right)^k e^{-\lambda\pi m^2} \frac{(\lambda\pi m^2)^k}{k!},$$

using the fact that ζ_m, conditioned on $\chi(B_m) = k$, is the sum of k independent and identically distributed terms. Thus

$$\varphi_m(y) = \sum_{k=0}^{\infty} \{\int_0^\infty \int_0^m e^{iyzf(r)} \frac{2r}{m^2} dr\, d F_Z(z)\}^k e^{-\lambda \pi m^2} \frac{(\lambda \pi m^2)^k}{k!}$$

$$= e^{-\lambda \pi m^2} \exp\{2\pi\lambda \int_0^\infty \int_0^m e^{iyzf(r)} r\, dr\, d F_Z(z)\}$$

$$= \exp\{2\pi\lambda \int_0^\infty \int_0^m \left(e^{iyzf(r)} - 1\right) r\, dr\, d F_Z(z)\}$$

$$\rightarrow \exp\{2\pi\lambda \int_0^\infty \int_0^\infty \left(e^{iyzf(r)} - 1\right) r\, dr\, d F_Z(z)\}$$

as $m \rightarrow \infty$, and the theorem is proved. In the following assume that the Z_j are bounded.

Corollary 1. The random variable ζ is finite a. s. if and only if

$$\int_0^\infty r\, f(r)\, dr < \infty. \tag{II-2}$$

Proof. From (II-1) it follows that $\zeta < \infty$ a. s. if and only if

$$\int_0^\infty \left(e^{iyzf(r)} - 1\right) r\, dr < \infty. \tag{II-3}$$

But

$$\left(e^{iyzf(r)} - 1\right) r = iyzf(r)\, r + r \cdot o\left(f(r)\right), \quad r \rightarrow \infty,$$

so that (II-3) holds if and only if (II-2) holds.

Corollary 2. If $\zeta < \infty$ a. s. and $\mu_Z = \int_0^\infty z\, d F_Z(z)$, then

$$E\, \zeta = 2\pi\lambda \mu_Z \int_0^\infty r\, f(r)\, dr < \infty, \tag{II-4}$$

and

$$\sigma_\zeta^2 = 2\pi\lambda\, E(Z^2) \int_0^\infty r\, f(r)^2\, dr < \infty.$$

Proof. This follows from (II-1) and the well-known formula $E\, \zeta^k = i^{-k} \varphi^{(k)}(0)$.

<u>Example</u>. Suppose N_W is of the form

$$N_W(r) = 10(\alpha \log r + \beta r), \quad r > r_0 \text{ (in yds)},$$

so that

$$f(r) = r^{-\alpha}\, 10^{-\beta r}, \quad r > r_0. \tag{II-5}$$

The parameter α is called the spreading constant and 10β is the attenuation coefficient. Then Corollary 1 implies that ambient noise is finite if and only if $\beta > 0$ or $\alpha > 2$. In particular, if one is modeling ambient noise and assuming spherical spreading or less ($\alpha \leq 2$) then one must consider the effects of attenuation (i.e., assume $\beta \neq 0$) or else spurious results will be obtained. This effect might not have been anticipated, especially for low frequency propagation, since attenuation at low frequencies is apparently insignificant. For example, attenuation is only about .0005 db/kyd at 50 Hz, and yet must be included in one's model if one is assuming an idealized infinite planar ocean. Of course if one is using a bounded noise process and the region has diameter say 3,000 miles, then attenuation can be safely ignored.

The mean value given by (II-4) can be evaluated in closed form for some cases of (II-5). Suppose f is of the form (II-5) with $r_0 = 1$ yd. Then

$$\int_0^\infty f(r)\, r\, dr \approx \int_1^\infty f(r)\, r\, dr = \int_1^\infty r^{-\alpha+1} e^{-\ln 10 \beta r} dr$$

$$= (\ln 10\beta)^{\alpha-2}\, \Gamma(-\alpha+2, \ln 10\beta),$$

where Γ is the incomplete gamma function, and the latter can be evaluated explicitly for $\alpha = 1, 3/2, 2$. Table II-1 gives some sample calculations.

At this point we should mention something about the need for having two mathematical models: the Poisson noise process and the bounded noise process. The main reason is that the Poisson noise process is much more suitable for proving theoretical results, but we need the bounded process to represent the real world. Moreover to simulate a sample path of the time series $t \to \eta(P, t)$ we can only generate a finite number of noise sources so the bounded noise process is required. Many of our conclusions are based on an examination of one of these processes and we will often glide effortlessly to the corresponding conclusion for the other process, when it seems plausible.

TABLE II-1

TYPICAL VARIATIONS IN AMBIENT NOISE AS A FUNCTION OF SPREADING CONSTANT

α	$E\zeta$	$E\eta \approx 10 \log E\zeta$ $\beta = .001$ db/nm
1	$2\pi\lambda\mu_Z \dfrac{e^{-\beta \ln 10}}{\beta \ln 10}$	$10 \log (2\pi\lambda\mu_Z) + 26.4$ db
	$\approx \dfrac{2\pi\lambda\mu_Z}{\beta \ln 10}$	
3/2	$2\pi\lambda\mu_Z \sqrt{\dfrac{\pi}{\beta \ln 10}}\,(1 - \Phi(\sqrt{\beta \ln 10}))$	$10 \log (2\pi\lambda\mu_Z) + 12.7$ db
	$\approx 2\pi\lambda\mu_Z \cdot \frac{1}{2}\sqrt{\dfrac{\pi}{\beta \ln 10}}$	
2	$2\pi\lambda\mu_Z(\beta \ln 10 - \ln\beta - \gamma - \ln\ln 10)$	$10 \log (2\pi\lambda\mu_Z) + 8.4$ db
	$\approx 2\pi\lambda\mu_Z \ln \beta^{-1}$	

CHAPTER III

THE AMBIENT NOISE TIME SERIES

In this chapter we study properties of the time series $t \to \eta(P, t)$, for a fixed P. Thus we suppress the dependence on P and write

$$\eta(t) = 10 \log \Sigma_j \, 10^{\left(N_j - N_W(r_j(t))\right)/10}$$

$$= 10 \log \Sigma_j \, Z_j \, f\left(r_j(t)\right),$$

where $r_j(t)$ is the distance of $S_j(t)$ from some fixed point P. In Chapter IV we will study the spatial function $P \to \eta(P, t)$, with t fixed. Recall that $S_j(t)$ is the position of the j^{th} noise source at time t. If η is a Poisson noise process, then the choice of P is immaterial and we assume it is the origin. If η is a bounded noise process then the choice of $P \in \mathcal{B}$ is relevant to the statistical behavior of the process $t \to \eta(P, t)$.

We will show that a bounded noise process <u>on a rectangle</u> $\mathcal{B}$ or a Poisson noise process is a stationary stochastic process. It is conjectured that an arbitrary bounded noise process is a stationary process but we have not been able to prove this. Also we discuss the sample path behavior of $t \to \eta(t)$, showing that this time series is approximately a Gauss-Markov process and giving a formula to approximate the relaxation time.

<u>Theorem</u>. The bounded noise process $t \to \eta(t)$ on a rectangle $\mathcal{B}$ is a stationary stochastic process.

<u>Proof</u>. We must show that the distribution of the random vector $(\eta(t_1+\Delta), \ldots, \eta(t_k+\Delta))$ is independent of Δ for any integer k, and time points $t_1, \ldots, t_k$. Since $\eta(t)$ depends deterministically on the random variables $S_1(t), \ldots, S_M(t), N_1, \ldots, N_M$, clearly it is sufficient to show that the distribution of the kM-dimensional vector

$$\left(S_1(t_1+\Delta), \ldots, S_1(t_k+\Delta), \ldots, S_M(t_1+\Delta), \ldots, S_M(t_k+\Delta)\right)$$

is independent of Δ. Moreover, since $S_i(t_l+\Delta)$ is independent of $S_j(t_m+\Delta)$ for any $i \neq j$, it suffices to show that the distribution of

$$\left(S_1(t_1+\Delta), \ldots, S_1(t_k+\Delta)\right)$$

is independent of Δ.

First we show that the distribution of $S_1(t)$ is uniform on $\mathscr{B}$ for any $t \geq 0$. Suppose without loss of generality that the vertices of $\mathscr{B}$ are at (0, 0), (a, 0), (a, b), (0, b), $a \geq b$. Let $h:(x, y) \to h(x, y)$ be the probability density of $S_1(t)$, $(x, y) \in \mathscr{B}$. Let $h(\cdot, \cdot \mid v_1)$ denote the conditional density of $S_1(t)$, conditioned on the velocity v_1. Then

$$h(x, y) = \int_0^\infty h(x, y \mid v_1)\, d F_V(v_1),$$

so it suffices to prove that $h(x, y \mid v_1)$ is uniform on $\mathscr{B}$ for a given fixed v_1. Suppose at first that $t < b/(2 v_1)$. We have

$$h(x, y \mid v_1) = \frac{1}{2\pi} \int_0^{2\pi} h(x, y \mid v_1, \theta_1)\, d\theta_1$$

where θ_1 is the initial heading of this noise source. Thus

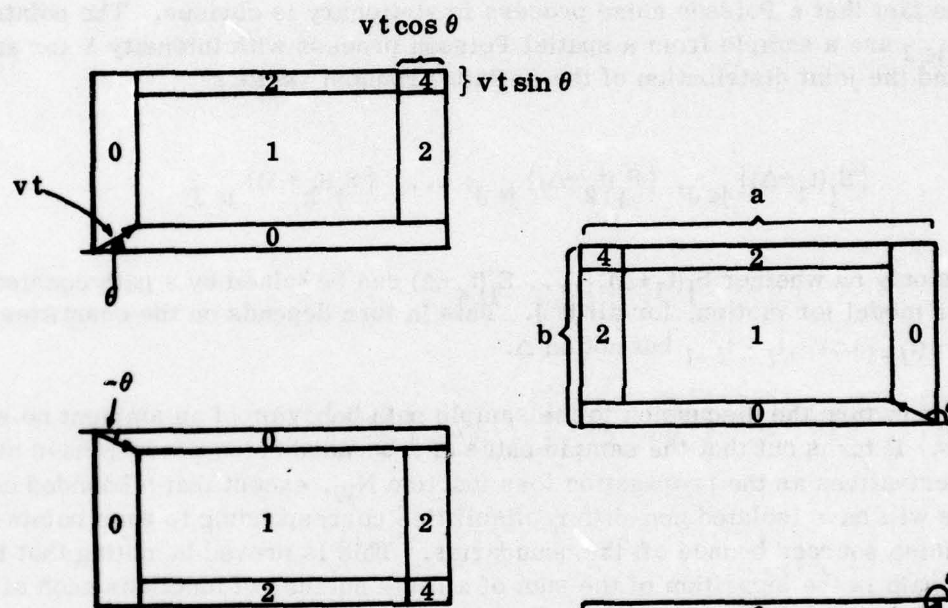
$$\begin{aligned} h(x, y \mid v_1) &= \frac{1}{2\pi}\left\{\left(\int_0^{\pi/2} + \int_{\pi/2}^{\pi} + \int_{\pi}^{3\pi/2} + \int_{3\pi/2}^{2\pi}\right) h(x, y \mid v_1, \theta_1)\, d\theta_1\right\} \\ &= \frac{1}{2\pi}\left\{\int_0^{\pi/2} h(x, y \mid v_1, \theta_1)\, d\theta_1 + \int_0^{\pi/2} h(x, y \mid v_1, \pi-\theta_1)\, d\theta_1\right. \\ &\qquad \left. + \int_0^{\pi/2} h(x, y \mid v_1, \theta_1+\pi)\, d\theta_1 + \int_0^{\pi/2} h(x, y \mid v_1, -\theta_1)\, d\theta_1\right\} \\ &= \frac{2}{\pi}\int_0^{\pi/2} \frac{1}{4}\left\{h(x, y \mid v_1, \theta_1) + h(x, y \mid v_1, \pi-\theta_1)\right. \\ &\qquad \left. + h(x, y \mid v_1, \theta_1+\pi) + h(x, y \mid v_1, -\theta_1)\right\} d\theta_1. \end{aligned}$$

A glance at Figure III-1 illustrates that the integrand in the last expression above is equal to $1/ab$ for any $t < b/(2 v_1)$.

FIGURE III-1

EFFECTS OF TRANSFORMATIONS IN DIRECTIONS θ, $-\theta$, $\pi-\theta$, $\theta+\pi$

Note: Number in each box is the value of the resulting density x a b.



Thus $S_1(t)$, conditioned on the value of v_1, is distributed uniformly for t sufficiently small and hence for any t, since the effect of an arbitrary t can be broken up into successive sufficiently small steps. Thus $S_1(t)$ is distributed uniformly, independent of t. Finally the joint distribution of $(S_1(t_1+\Delta), S_1(t_2+\Delta), \ldots, S_1(t_k+\Delta))$ depends only on whether the given points can be joined by an admissible path and on the speed v_1 required to traverse this path, and not on Δ. This completes the proof.

The details of the proof make clear that to extend this result to an arbitrary bounded noise process one need only show that our model for motion preserves the uniform distribution on the region $\mathscr{B}$. Our method of proving this for a rectangular $\mathscr{B}$ obviously will not apply to a general region which does not have the symmetries of a rectangle.

The fact that a Poisson noise process is stationary is obvious. The points $\{S_j(t)\}_{j\in J}$ are a sample from a spatial Poisson process with intensity λ for any $t \geq 0$, and the joint distribution of the (infinite) random vectors

$$\{S_j(t_1+\Delta)\}_{j\in J}, \{S_j(t_2+\Delta)\}_{j\in J}, \ldots, \{S_j(t_k+\Delta)\}_{j\in J}$$

depends only on whether $S_j(t_1+\Delta), \ldots, S_j(t_k+\Delta)$ can be joined by a path consistent with our model for motion, for all $j \in J$. This in turn depends on the quantities $(t_l+\Delta) - (t_{l-1}+\Delta) = t_l - t_{l-1}$ but not on Δ.

We now turn the discussion to the sample path behavior of an ambient noise process. It turns out that the sample paths of a bounded noise process have as many derivatives as the propagation loss function N_W, except that a bounded noise process will have isolated non-differentiabilities corresponding to time points at which noise sources bounce off the boundaries. This is proved by noting that the sample path is the logarithm of the sum of a finite number of functions each of which has the required smoothness. If N_W is differentiable, $\int r f'(r)\, dr < \infty$, and both Z_j and v_j have finite means, then the corresponding Poisson noise process sample paths are differentiable with no exceptions since the noise sources do not bounce off the boundaries. This follows from examining the difference quotient

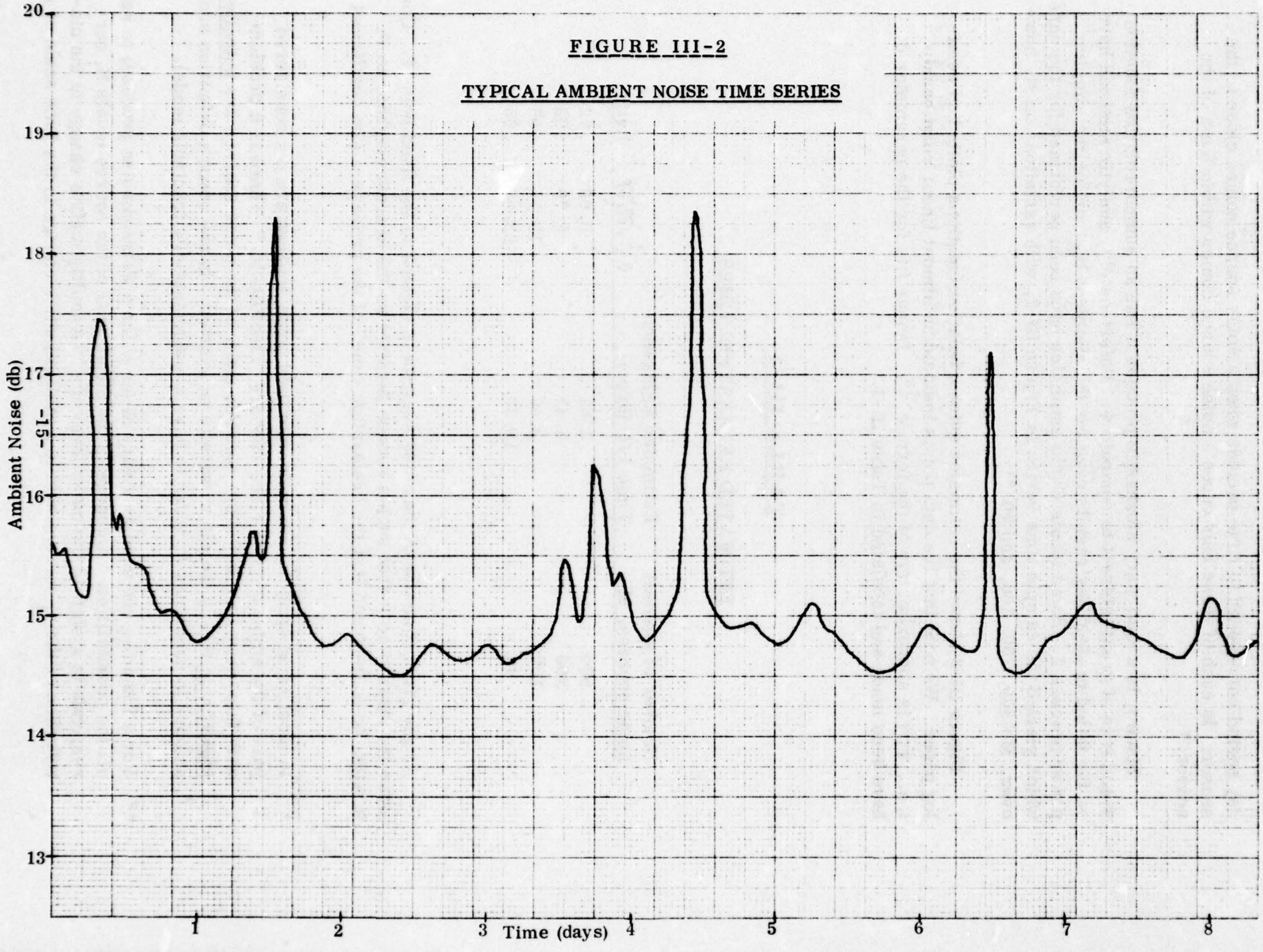
$$h^{-1}\left(\xi(t+h) - \xi(t)\right) = h^{-1} \Sigma Z_j\left[f\left(r_j(t+h)\right) - f\left(r_j(t)\right)\right]$$

$$\rightarrow \Sigma Z_j f'\left(r_j(t)\right) r_j'(t) \text{ as } h \rightarrow 0.$$

Figure III-2 shows a sample path for a bounded noise process on a square region $\mathscr{B}$ with area $A = 9 \times 10^6$ square miles, with $M = 900$ noise sources, the v_j's uniformly distributed between 15 and 25 knots, $N_j = 180$ db//1μ Pa//1 Hz, and propagation loss 1A from Chapter IV. The four large excursions during this eight

FIGURE III-2

TYPICAL AMBIENT NOISE TIME SERIES



day period correspond to time periods when a noise source comes close to the sensor. In each of these four cases a noise source comes within 6 nm of the sensor. *

Since η is a stationary stochastic process it has an autocorrelation function Ψ and it is of great interest to compute Ψ. Unfortunately, analytic methods have so far failed to yield any results even for the simplest N_W, or for the simpler power process ζ. Some Monte Carlo estimates have been performed for the case which yielded the sample time series in Figure III-2, with variations on M, however, M = 900, 300, 100, and 40. **

Figure III-3 shows the estimated autocorrelation functions plotted on semi-log paper. We note that the data points indicate an almost linear relationship, i.e., $\Psi(t)$ is approximately of the form $e^{-\delta t}$. Linear fits for the parameter δ have been made and tabulated in Table III-1.

TABLE III-1

ESTIMATED RELAXATION TIMES

Number of ambient noise sources, M	Estimated Relaxation Time $1/\delta$ (hrs.)	$\bar{v}^{-1}\sqrt{A/M}$	Ratio
900	3.61	5.00	.72
300	5.41	8.70	.62
100	8.47	15.15	.56
40	15.38	23.80	.65

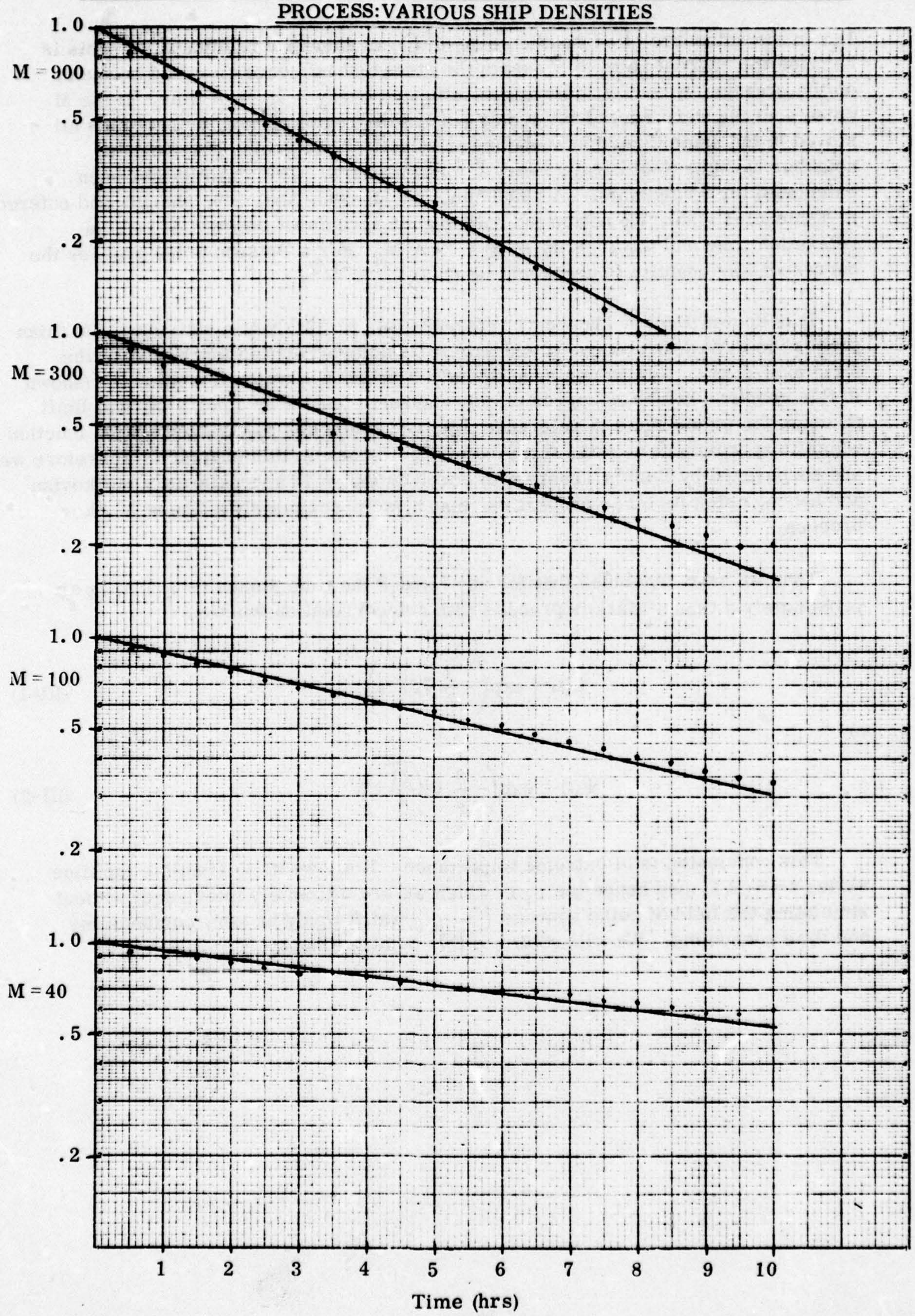
At any given time one of the noise sources is closest to the fixed point P. Consider the mean time ν that such a source persists as the nearest neighbor to P. It might be conjectured that the "relaxation time" of the process η (the coefficient

* In most naval applications such near field noise sources are not considered part of the ambient noise since they are "identifiable." There is a problem with this viewpoint, however, since so many of the noise sources are potentially identifiable if only the effort were to be exerted. In any event a definition such as ours is necessary in order to have a mathematically tractable model.

** For a fixed N_W one need only vary M (or λ for a Poisson noise process) to see all the possibilities. A change in the mean level of the source levels N_j corresponds to a shift in the mean level of η. A multiplicative change to the distribution of the v_j corresponds to a multiplicative change in the time scale of η.

FIGURE III-3

ESTIMATED AUTOCORRELATION FUNCTIONS FOR THE AMBIENT NOISE PROCESS: VARIOUS SHIP DENSITIES



$1/\delta$ in the autocorrelation function $\exp(-\delta\,|t|)$) is somehow related to ν. This is because the nearest point to P exerts the strongest influence on η and because, on the average, η comes under a new influence every ν units of time. If the M noise sources were spread out evenly across the region $\mathcal{B}$ and the particles all moved in the same direction at the mean velocity $\bar{v} = 20$ knots, then nearest neighbor changes would occur every $\bar{v}^{-1}\sqrt{A/M}$ hours. This quantity has been calculated for $\bar{v} = 20$ knots, $\sqrt{A} = 3000$ nm, and M = 900, 300, 100, and 40, and entered in Table III-1 also. We notice the rather good agreement between estimated relaxation time and the quantity $(2/3)\,\bar{v}^{-1}\sqrt{A/M}$. For a Poisson noise process the corresponding quantity to use would be $(2/3)\,\bar{v}^{-1}\,\lambda^{-1/2}$.

It is known (Doob's Theorem, reference [g], p. 321) that a stationary Gaussian process is a Markov process if and only if its autocorrelation function is of the form $\exp(-\delta\,|t|)$. Now in our case we have a process η which is stationary (shown in this chapter), which is approximately Gaussian (shown by using a central limit theorem for power sums in reference [h]), and which has an autocorrelation function which is approximately of the form $\exp(-\delta\,|t|)$ (shown in this chapter). Therefore we are encouraged by Doob's Theorem to conclude that η is approximately Markovian and hence, using its other properties, that η is approximately a Gauss-Markov process.

Thus we have concluded that for any fixed P the time series $t \rightarrow \eta(P,t)$ is approximately a Gauss-Markov process with autocorrelation function

$$\Psi(t) = \exp\left(-\frac{3}{2}\,\bar{v}\sqrt{\lambda}\;t\right) \qquad \text{(III-1)}$$

or

$$\Psi(t) = \exp\left(-\frac{3}{2}\,\bar{v}\sqrt{\frac{M}{A}}\;t\right). \qquad \text{(III-2)}$$

This conclusion is of extreme importance. It allows us to simulate the time series $t \rightarrow \eta(P,t)$ (and hence the η_i in which we are ultimately interested) without simulating the field of noise sources $\{S_j\}_{j\in J}$ which would be very cumbersome and time consuming. We will return to this point in Chapter V.

CHAPTER IV

SPATIAL CORRELATION OF AMBIENT NOISE

In this chapter we look at the process $P \rightarrow \eta(P, t)$ at a fixed time t,

$$\eta(P) = 10 \log \Sigma_j \, 10^{\left(N_j - N_W\left(r_j(P)\right)\right)/10}$$

$$= 10 \log \Sigma_j Z_j \, f\left(r_j(P)\right)$$

$$= 10 \log \zeta(P),$$

where $r_j(P)$ is the distance between the j^{th} noise source and the point P, at some fixed time t.

For a Poisson noise process η is a stationary process as a function of P, but no bounded noise process can be stationary. For the rest of this chapter we assume η is a Poisson noise process.

An analytic expression for the correlation between $\zeta(P)$ and $\zeta(Q)$ can be given.

Theorem. Let $P = (0, 0)$ and $Q = (u, 0)$. Then the correlation ρ between $\zeta(P)$ and $\zeta(Q)$ is given by

$$\rho = \frac{\int_0^{2\pi}\int_0^{\infty} r\, f(r)\, f\left((r^2 + u^2 - 2ru\cos\theta)^{\frac{1}{2}}\right) dr\, d\theta}{2\pi \int_0^{\infty} r\, f(r)^2\, dr}. \qquad \text{(IV-1)}$$

Proof. Let r_j and r_j' be the distances of the j^{th} point (x_j, y_j) of the Poisson sample from the points P and Q respectively. Let B_n be the disk with radius n centered at the origin. Let

$$\zeta_n = \sum_{(x_j, y_j) \in B_n} Z_j \, f(r_j)$$

and

$$\zeta_n' = \sum_{(x_j, y_j) \in B_n} Z_j f(r_j')$$

so that $\zeta_n \to \zeta(P)$ a.s. and $\zeta_n' \to \zeta(Q)$ a.s., and hence

$$E\,\zeta_n\,\zeta_n' \to E\,\zeta(P)\,\zeta(Q), \text{ as } n \to \infty.$$

Lex χ_n be the number of points from the Poisson sample in the set B_n. Then χ_n is a random variable with mean $\lambda\pi n^2$. Let J_n be the subset of indices j such that $(x_j, y_j) \in B_n$. Then

$$E\,\zeta_n\,\zeta_n' = E \sum_{j \in J_n} Z_j f(r_j) \sum_{l \in J_n} Z_l f(r_l')$$

$$= E \sum_{j \in J_n} Z_j^2 f(r_j) f(r_j')$$

$$+ E \sum_{j \in J_n} Z_j f(r_j) \sum_{\substack{l \in J_n \\ l \neq j}} Z_l f(r_l')$$

$$= E(\chi_n)\, E(Z^2) \frac{1}{\pi n^2} \int_0^{2\pi}\int_0^n f(r)\, f\left((r^2 + u^2 - 2ru\cos\theta)^{\frac{1}{2}}\right) r\,dr\,d\theta$$

$$+ \sum_{k=1}^{\infty} E\left\{ \sum_{j \in J_n} Z_j f(r_j) \sum_{\substack{l \in J_n \\ l \neq j}} Z_l f(r_l') \,\middle|\, \chi_n = k \right\} \Pr(\chi_n = k)$$

$$= \lambda\, E(Z^2) \int_0^{2\pi}\int_0^n f(r)\, f\left((r^2 + u^2 - 2r\cos\theta)^{\frac{1}{2}}\right) r\,dr\,d\theta$$

$$+ \sum_{k=1}^{\infty} k(k-1) \Pr(\chi_n = k)\, (EZ)^2 \left(\frac{2}{n^2} \int_0^n f(r)\, r\,dr \right)$$

$$\times \frac{1}{\pi n^2} \int_0^{2\pi}\int_0^n f\left((r^2 + u^2 - 2ru\cos\theta)^{\frac{1}{2}}\right) r\,dr\,d\theta$$

$$= \lambda\, E(Z^2) \int_0^{2\pi}\int_0^n f(r)\, f\left((r^2+u^2-2ru\cos\theta)^{\frac{1}{2}}\right) r\, dr\, d\theta$$

$$+ (\lambda \pi n^2)^2 (EZ)^2 \left(\frac{2}{n^2}\int_0^n f(r)\, r\, dr\right)$$

$$\times \frac{1}{\pi n^2} \int_0^{2\pi}\int_0^n f\left((r^2+u^2-2ru\cos\theta)^{\frac{1}{2}}\right) r\, dr\, d\theta$$

$$\to \lambda\, E(Z^2) \int_0^{2\pi}\int_0^\infty f(r)\, f\left((r^2+u^2-2ru\cos\theta)^{\frac{1}{2}}\right) r\, dr\, d\theta$$

$$+ \left[2\lambda\pi\, E(Z) \int_0^\infty f(r)\, r\, dr\right]^2.$$

Here we have used the fact that given there are χ_n points in B_n, they are distributed independently and uniformly. The formula (IV-1) now follows since the bracketed quantity above is none other than $E\,\xi(P)$ and since $\sigma^2_{\xi(P)} = 2\pi\lambda\, E(Z^2)\int_0^\infty r\, f(r)^2\, dr$.

We note that correlation is independent of λ and of the Z distribution, depending only on the propagation loss.

An approximate formula for the correlation $\mathscr{C}(u)$ between $\eta(P)$ and $\eta(Q)$, P and Q separated by distance u, can be given which we will use to show that the function $\mathscr{C}$ is also approximately independent of λ and the Z distribution.

Suppose X and Y have a bivariate Gaussian distribution, each with mean m and variance τ^2 and correlation ρ_0. Then elementary computations show that the variables $10^{X/10}$ and $10^{Y/10}$ have mean μ, variance σ^2, and correlation ρ given by

$$\mu = \exp\left(\frac{\ln 10}{10}\, m + \frac{1}{2}\left(\frac{\ln 10}{10}\right)^2 \tau^2\right),$$

$$\sigma^2 = \left[\exp\left(\frac{\ln 10}{5}\, m + \left(\frac{\ln 10}{10}\right)^2 \tau^2\right)\right]\left[\exp\left(\left(\frac{\ln 10}{10}\right)^2 \tau^2\right) - 1\right],$$

and

$$\rho = \left[\exp\left(\rho_0\left(\frac{\ln 10}{10}\right)^2 \tau^2\right) - 1\right]\left[\exp\left(\left(\frac{\ln 10}{10}\right)^2 \tau^2\right) - 1\right]^{-1}$$

$$= \left[\exp\left(\rho_0\left(\frac{\ln 10}{10}\right)^2 \tau^2\right) - 1\right]\frac{\mu^2}{\sigma^2}.$$

Solving for ρ_0 we find

$$\rho_0 = \frac{\ln(1+\rho\sigma^2/\mu^2)}{\ln(1+\sigma^2/\mu^2)}. \tag{IV-2}$$

But, as has already been noted, η is approximately Gaussian and so the correlation between $\eta(P)$ and $\eta(Q)$ is given by (IV-2) where ρ is the correlation between $\zeta(P)$ and $\zeta(Q)$ and is given in (IV-1). The quantity σ^2/μ^2 appearing in (IV-2) is given by

$$\frac{\sigma^2}{\mu^2} = \frac{E(Z^2)}{2\pi\lambda(EZ)^2} \frac{\int_0^\infty f(r)^2 \, r \, dr}{\left(\int_0^\infty f(r) \, r \, dr\right)^2}, \tag{IV-3}$$

using Corollary 2 of Chapter I. If f is of the form (II-5) with $\alpha = 1$, for example*, then

$$\frac{\sigma^2}{\mu^2} \approx \frac{E(Z^2)}{2\pi\lambda(EZ)^2} (\ln 10)^2 \beta^2 \ln(2\beta)^{-1} \tag{IV-4}$$

$$= \lambda^{-1} \frac{E(Z^2)}{(EZ)^2} \times 5 \times 10^{-6},$$

for $\beta = .001$ db/nm. Now for reasonable values of λ and $E(Z^2)/(EZ)^2$ such as $\lambda > .0001$ and $E(Z^2)/(EZ)^2 < 2$, we will have $\sigma^2/\mu^2 << 1$, and

$$\mathscr{C}(u) \approx \frac{\ln\left(1+\frac{\sigma^2}{\mu^2}\rho\right)}{\ln\left(1+\frac{\sigma^2}{\mu^2}\right)} \approx \rho.$$

Thus, as asserted, the correlation between $\eta(P)$ and $\eta(Q)$ depends, to a good approximation, only on propagation loss.

This fact is extremely useful, since it allows us to make predictions concerning spatial correlation of ambient noise based only on propagation loss in an area, with no need for predictions on the density or noisiness of the traffic. If a Monte Carlo simulation is employed one need only plug in some density λ and constant noise Z and estimate correlation for a given f; there is no need to vary λ and F_Z to see the effects of such variation.

---

* The only value of α for which the calculation (IV-3) can be performed in closed form is $\alpha = 1$, and hopefully the approximations to follow are still close for $1 < \alpha \leq 2$.

In principle (IV-1) can be computed by numerical analysis and the approximation (IV-2) employed, with σ^2/μ^2 given by (IV-3). However, it is easier to estimate $\mathscr{C}$ as a function of spatial separation via a simulation, and thereby relieve any misgivings about the approximation (IV-2), and avoid the difficult numerical analysis involved in (IV-1).

Figures IV-1 to IV-4 show some spatial correlation functions $\mathscr{C}$ corresponding to the potpourri of propagation loss functions graphed in Figures IV-5 to IV-12. In each case a density of .0001 ships/(nm)^2 with source level 180 db was assumed. For ranges past 250 nm propagation loss was assumed to be of the form $b + 12.2 \log(2000r)$, r in nm, the constant b chosen to make propagation loss continuous at $r = 250$ nm. In each case $\mathscr{C}(u)$ was estimated for 14 values of u ranging from 1 to 75 nm, 1000 replications in each case.

The main conclusions noted from an examination of these examples are:

(1) correlation is significant (say greater than .3) out to a range, depending on propagation loss, that may be anywhere from 10 to 100 nm,

(2) convergence zones induce ripples in the correlation function, and,

(3) CZ's induce a psuedo-randomness in the sense that if the oscillations in propagation loss caused by CZ's have large amplitude, then correlation goes to zero quickly.

The last point makes sense upon reflection: a slight change in the position of a noise source may alter its effects on a sensor from something significant to something insignificant and vice versa. Thus the amount of coherence in the noise experienced at two points becomes less predictable, i.e., correlation goes down.

FIGURE IV-1

CORRELATION FUNCTIONS CORRESPONDING TO ENVIRONMENTS 1A AND 1B

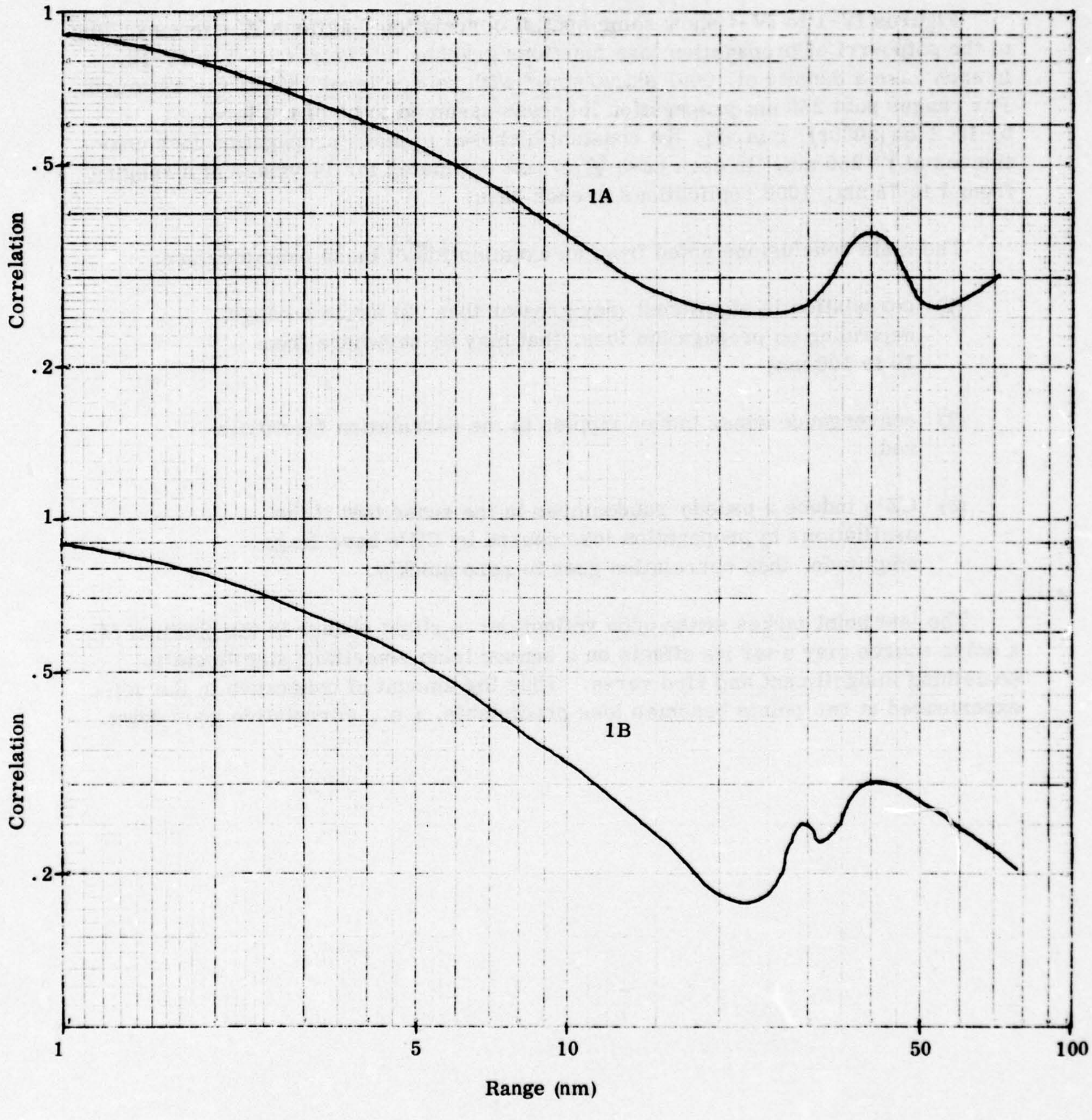


FIGURE IV-2

CORRELATION FUNCTIONS CORRESPONDING TO ENVIRONMENTS 2A AND 2B

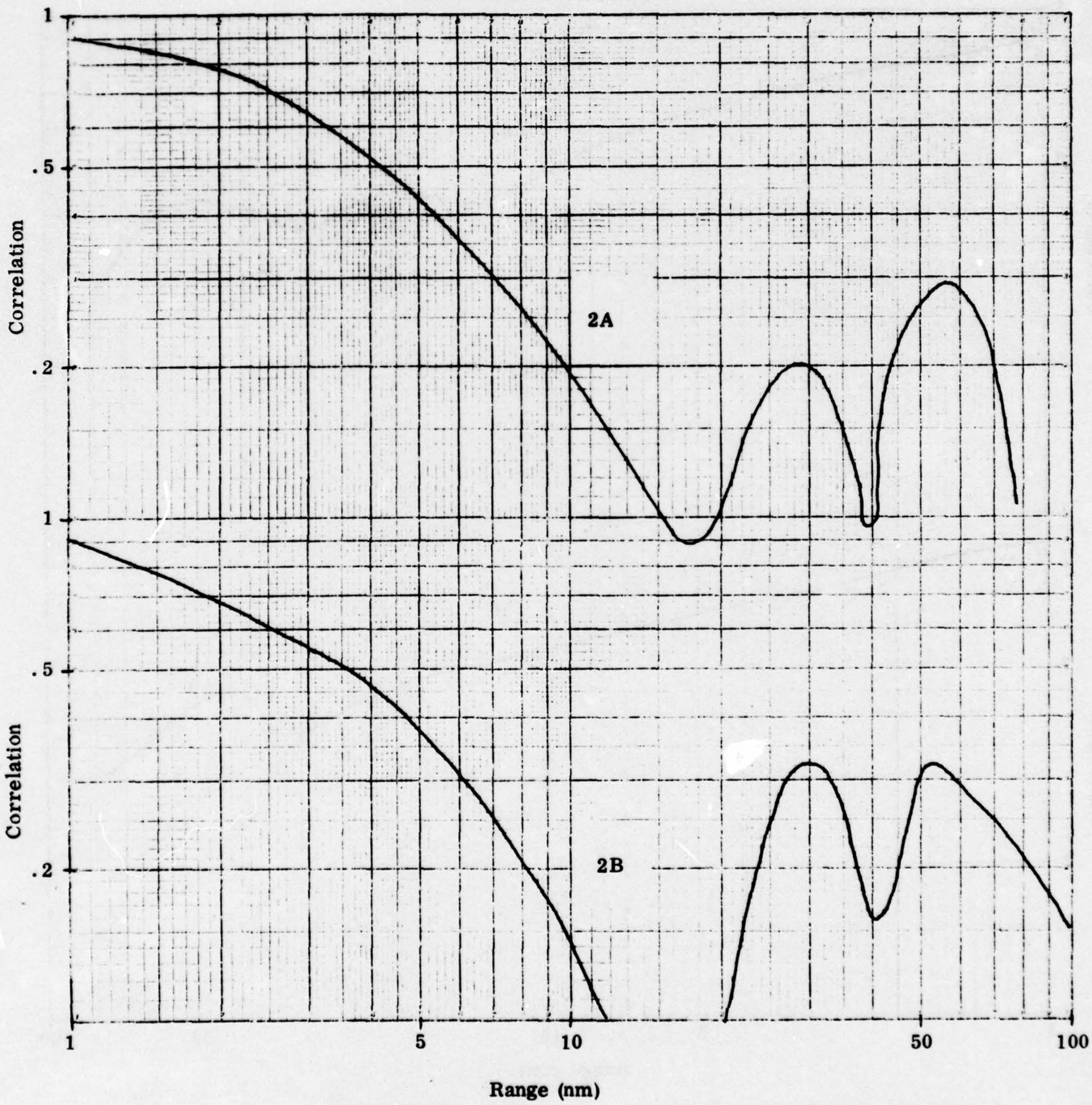


FIGURE IV-3

CORRELATION FUNCTIONS CORRESPONDING TO ENVIRONMENTS 3A AND 3B

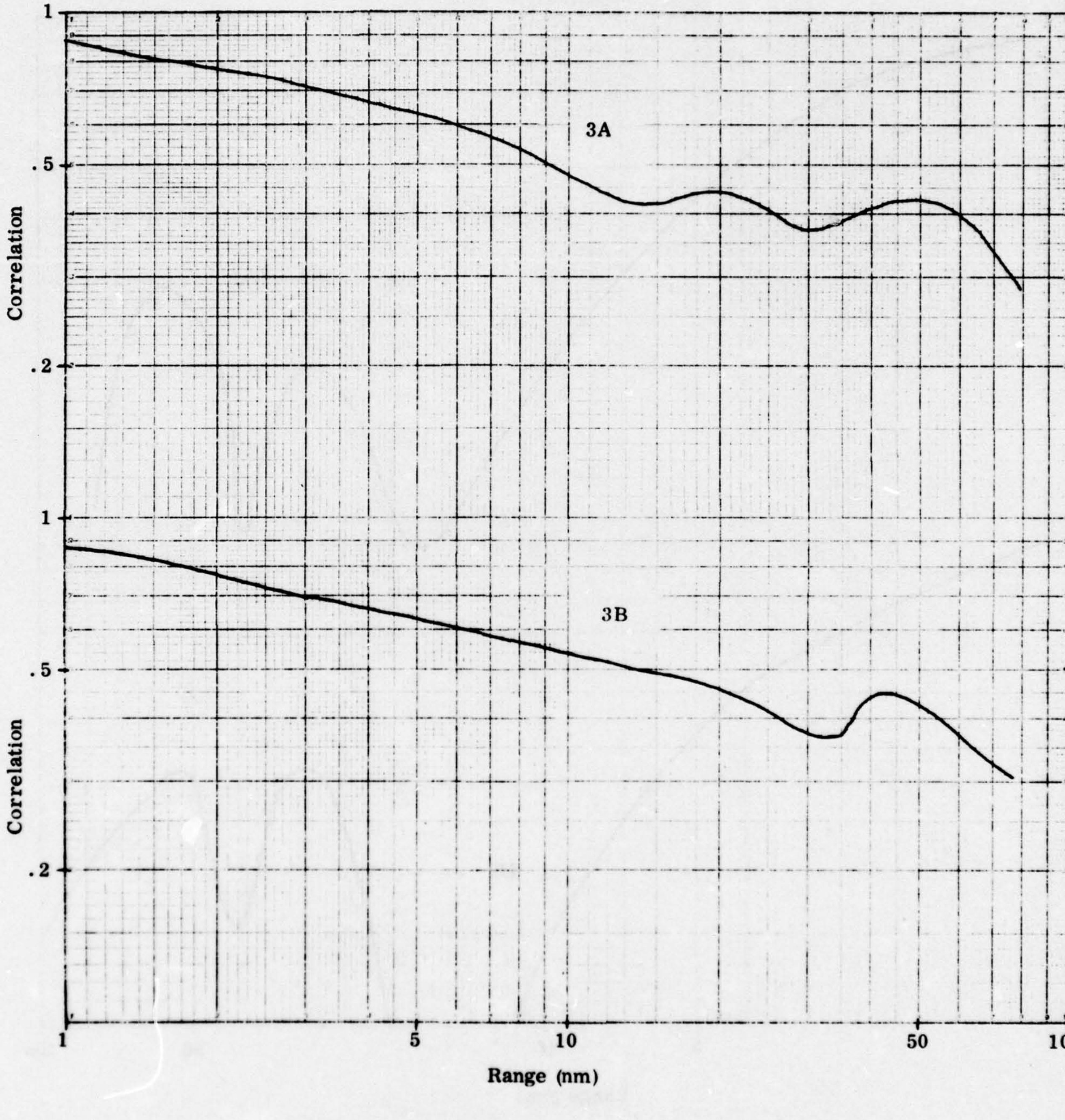


FIGURE IV-4

CORRELATION FUNCTIONS CORRESPONDING TO ENVIRONMENTS 4A AND 4B

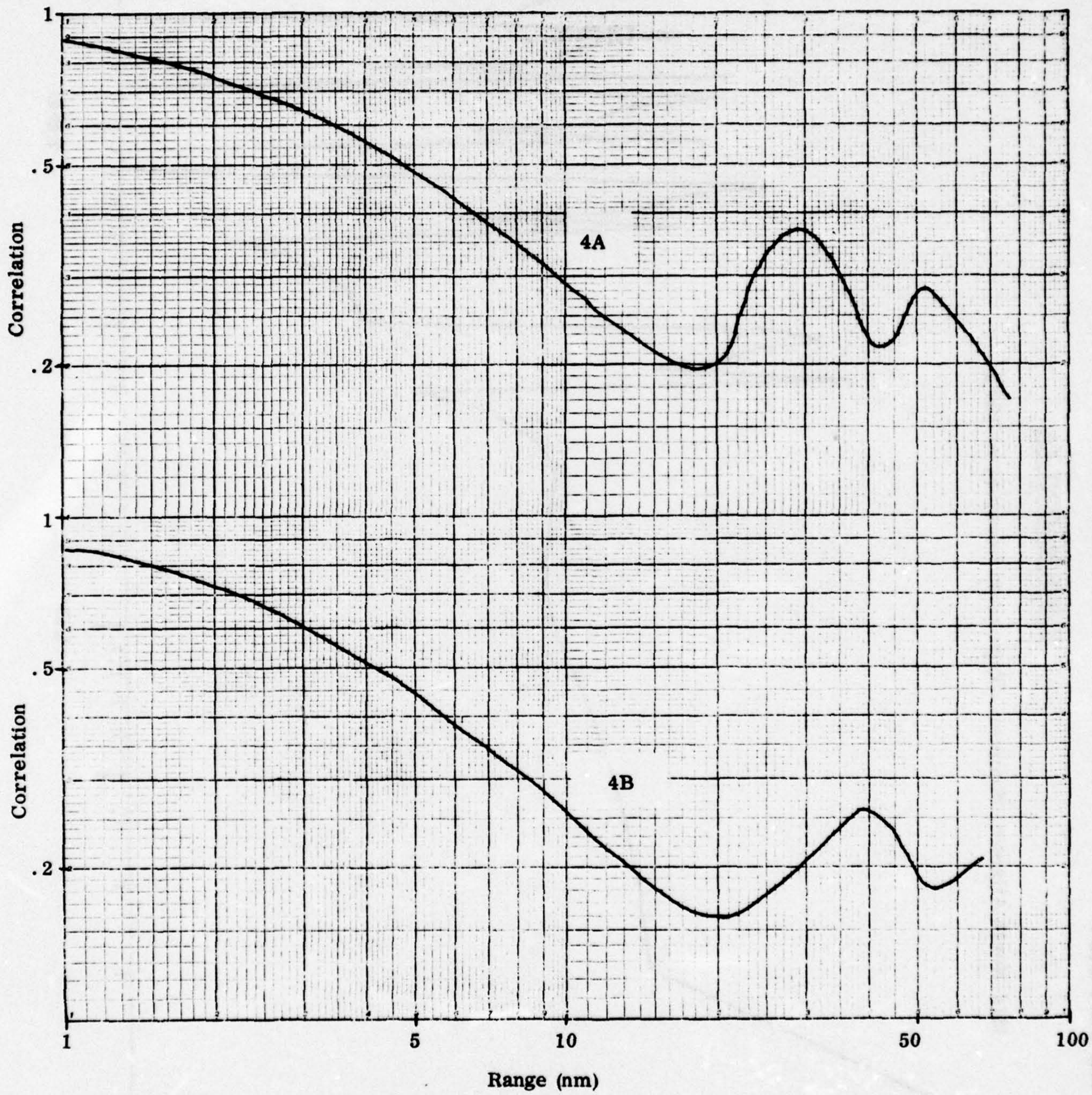


FIGURE IV-5

ENVIRONMENT 1A

Mid-North Atlantic, Summer, Source and Receiver at 150 ft.

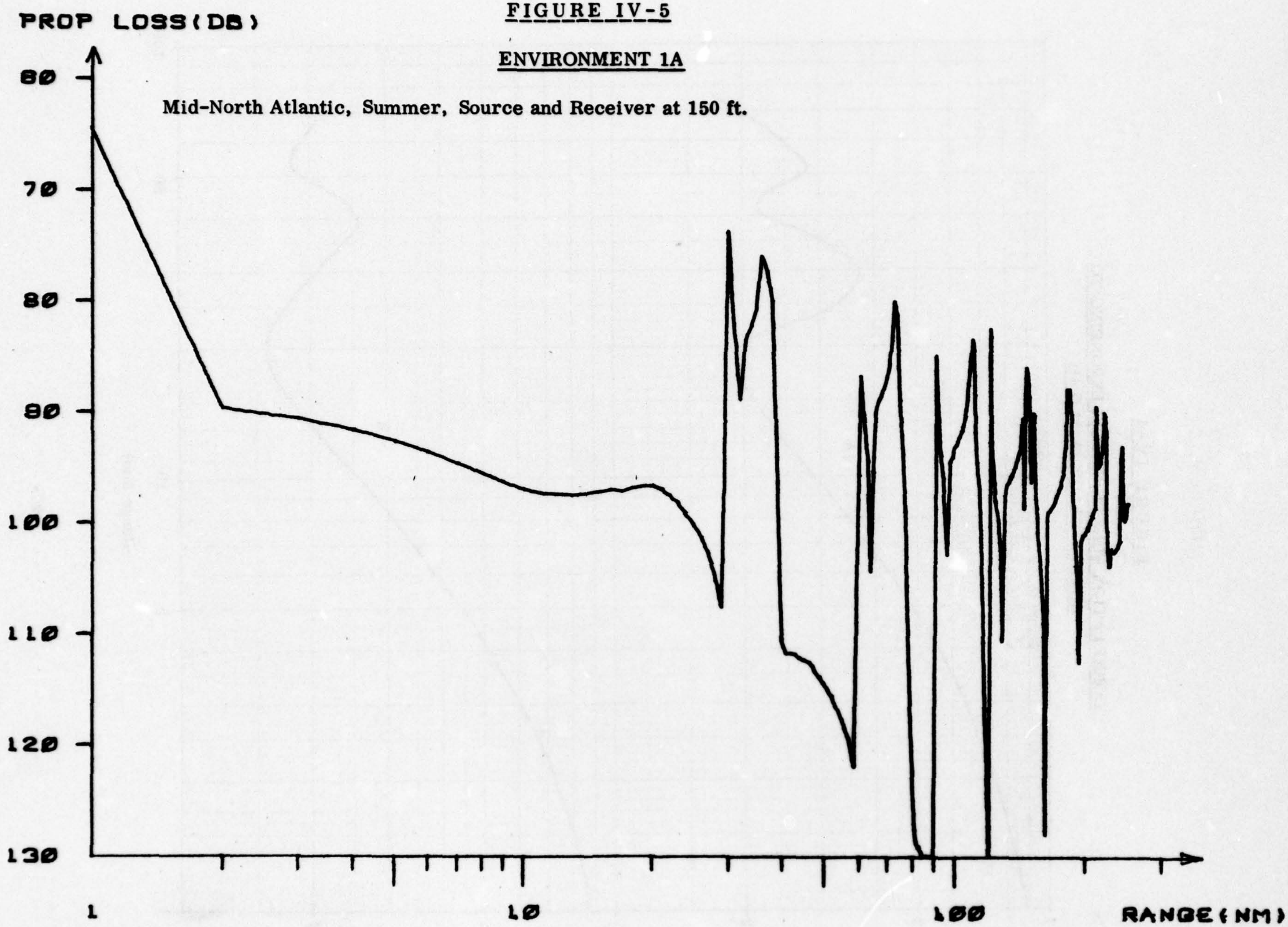


FIGURE IV-6

ENVIRONMENT 1B

Mid-North Atlantic, Summer, Source and Receiver at 150 ft.

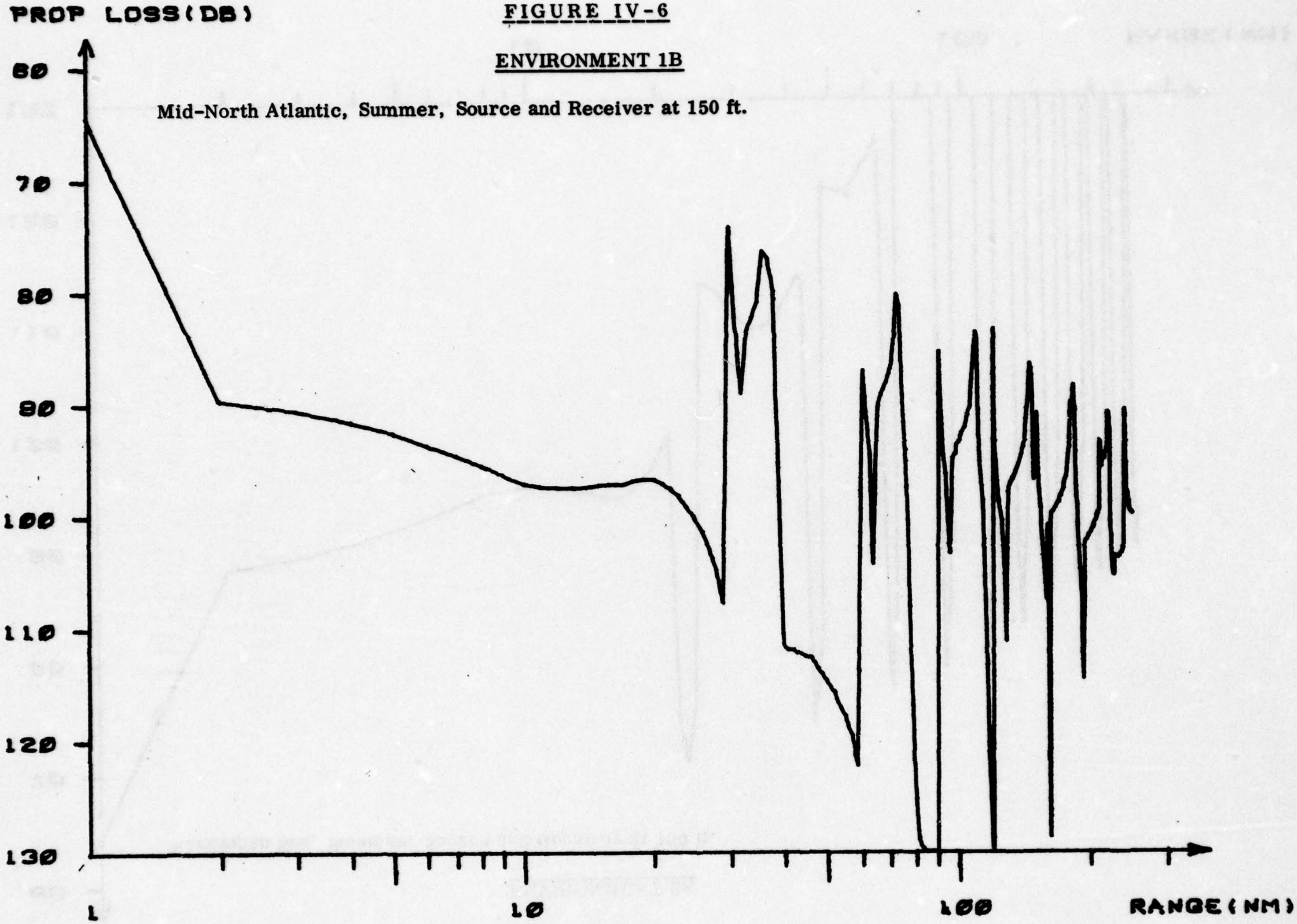


FIGURE IV-7

ENVIRONMENT 2A

Norwegian Sea, Summer, Source and Receiver at 150 ft.

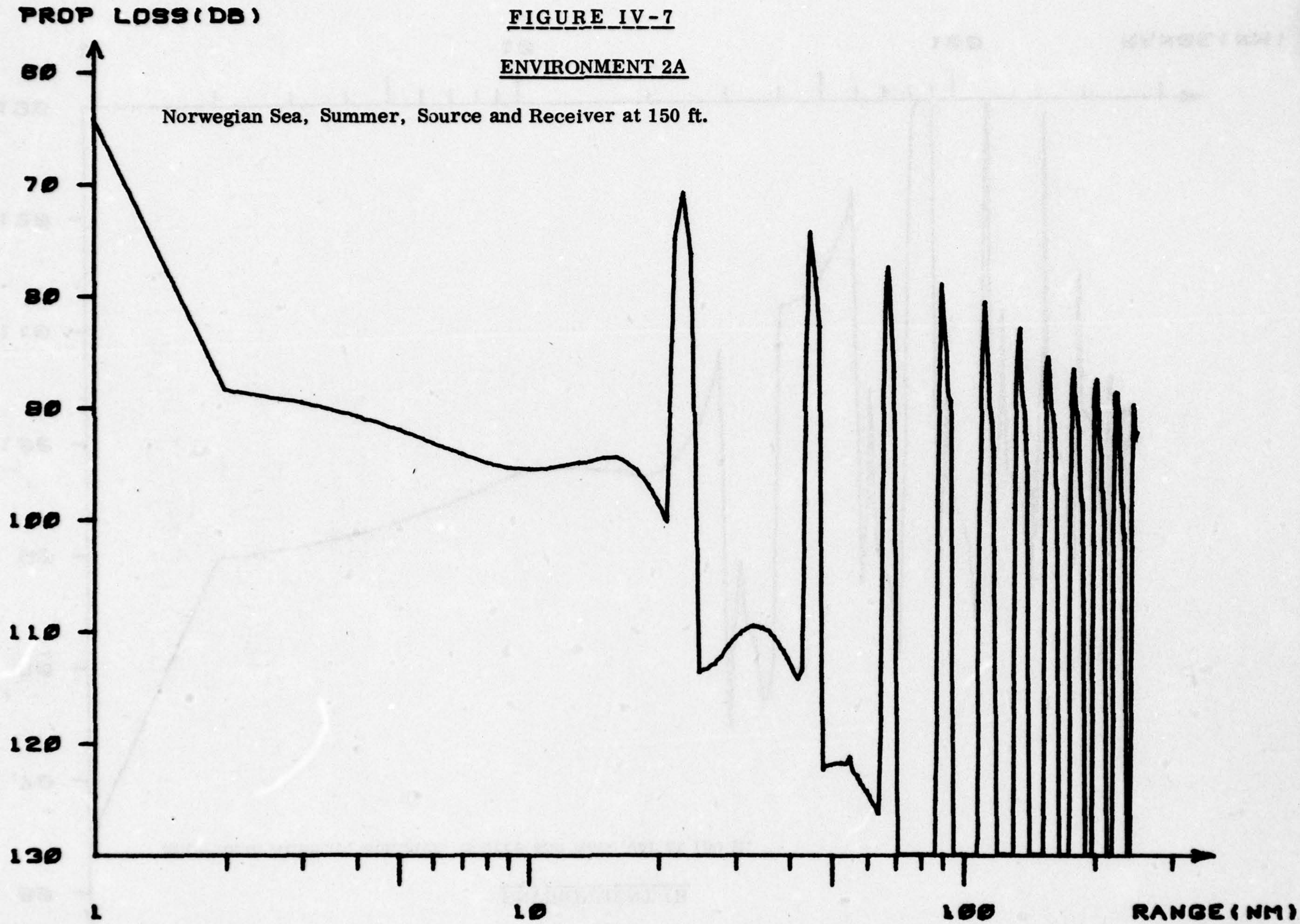


FIGURE IV-8

ENVIRONMENT 2B

Norwegian Sea, Summer, Source and Receiver at 150 ft.

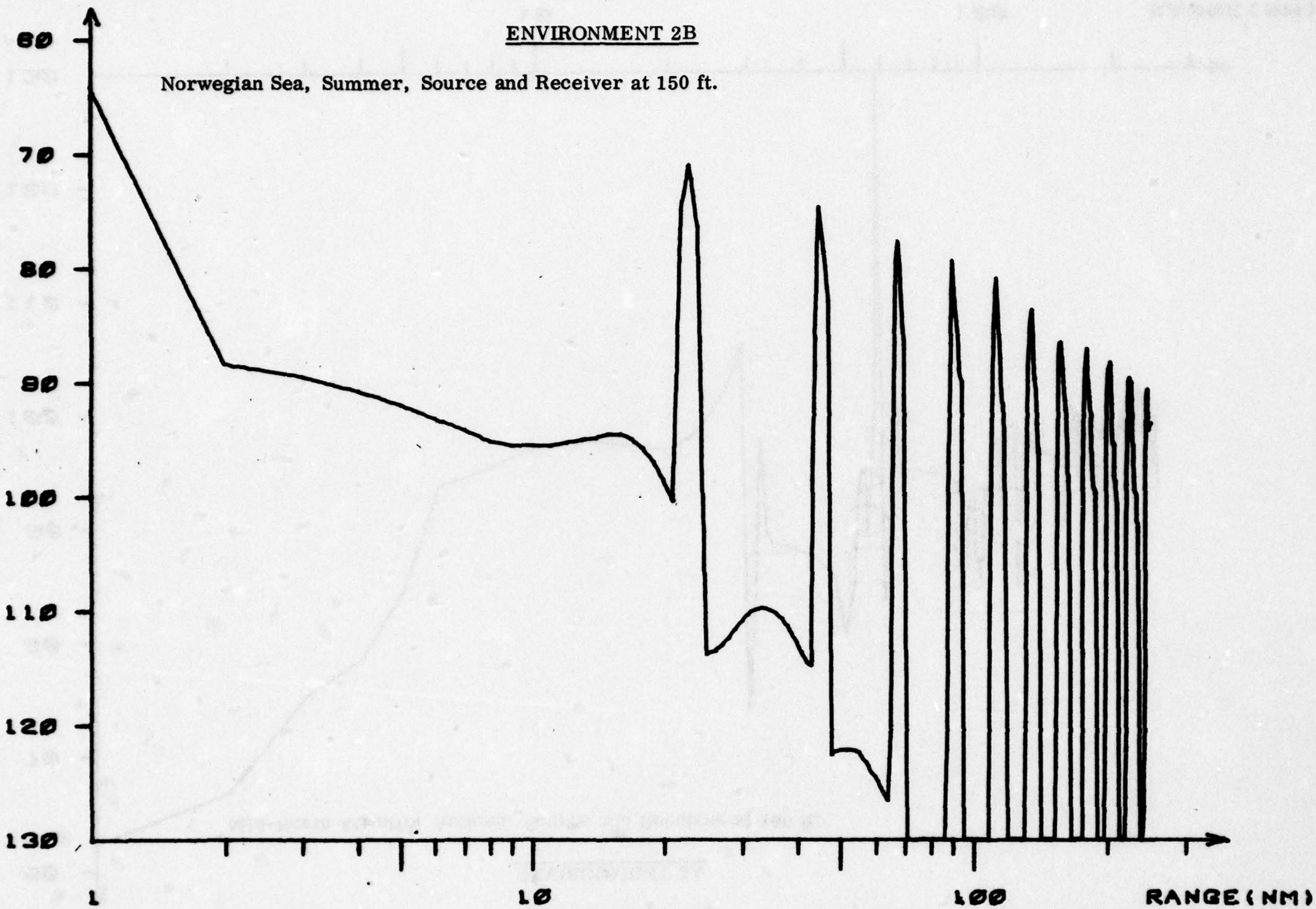
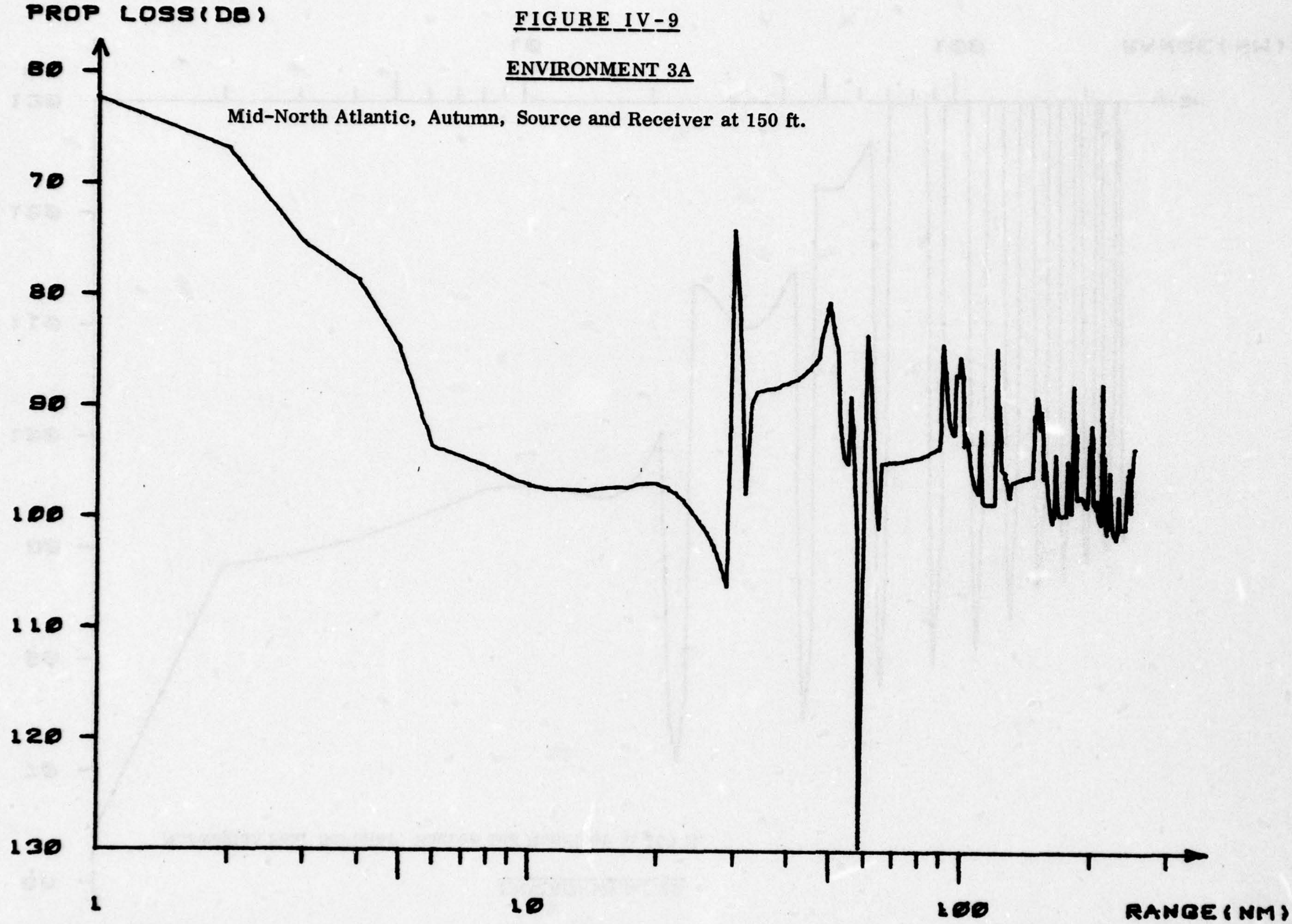


FIGURE IV-9

ENVIRONMENT 3A

Mid-North Atlantic, Autumn, Source and Receiver at 150 ft.



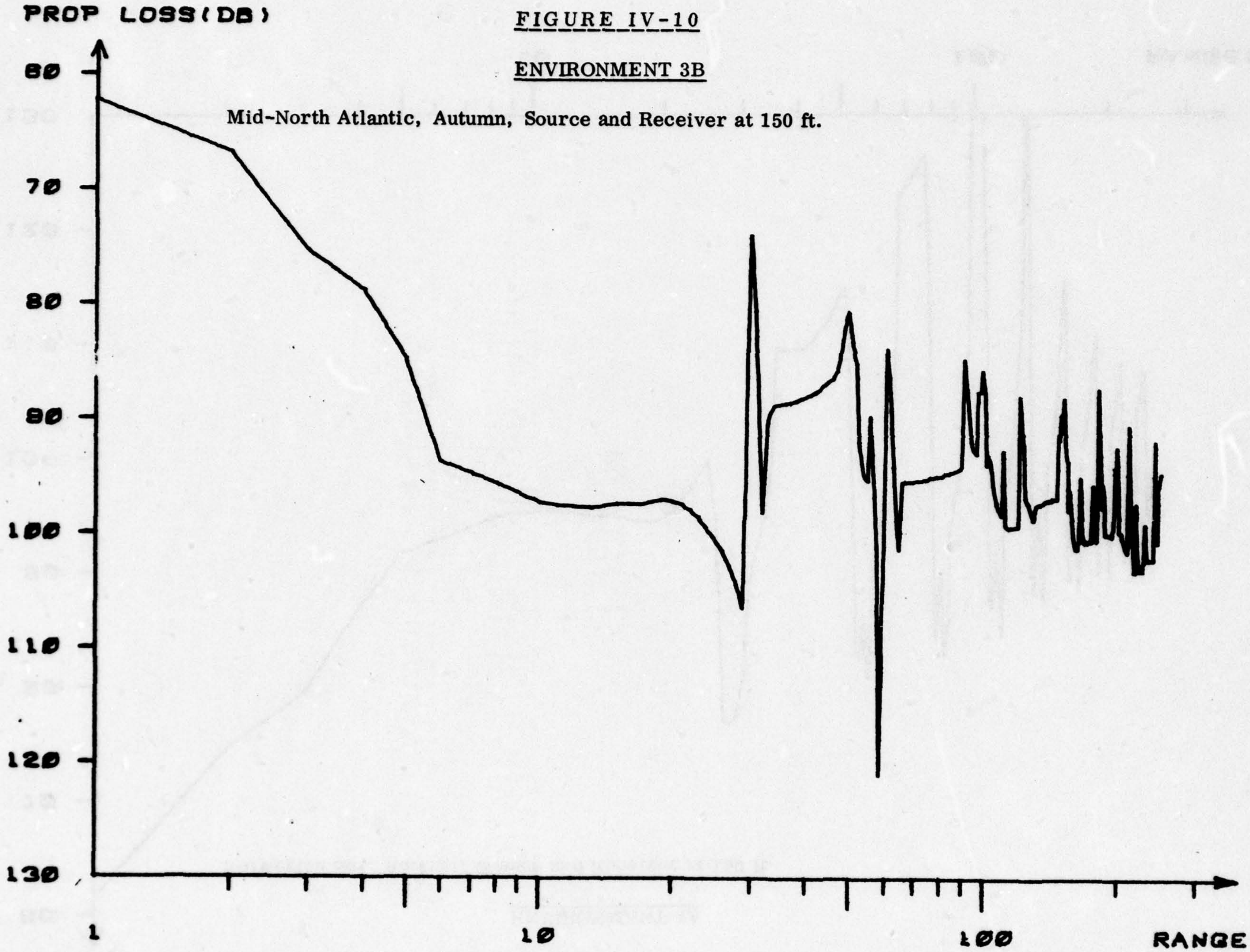


FIGURE IV-10

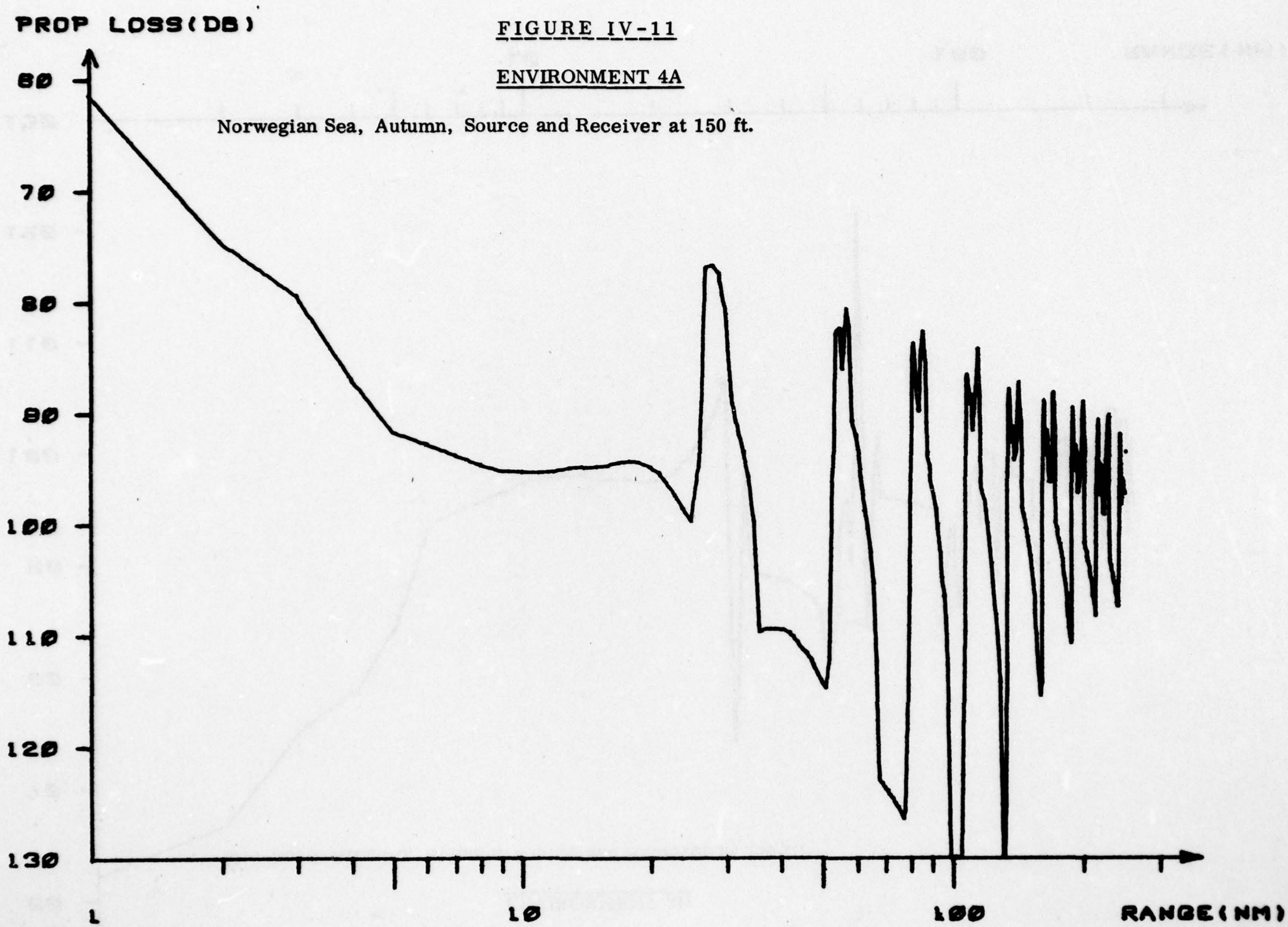
ENVIRONMENT 3B

Mid-North Atlantic, Autumn, Source and Receiver at 150 ft.

FIGURE IV-11

ENVIRONMENT 4A

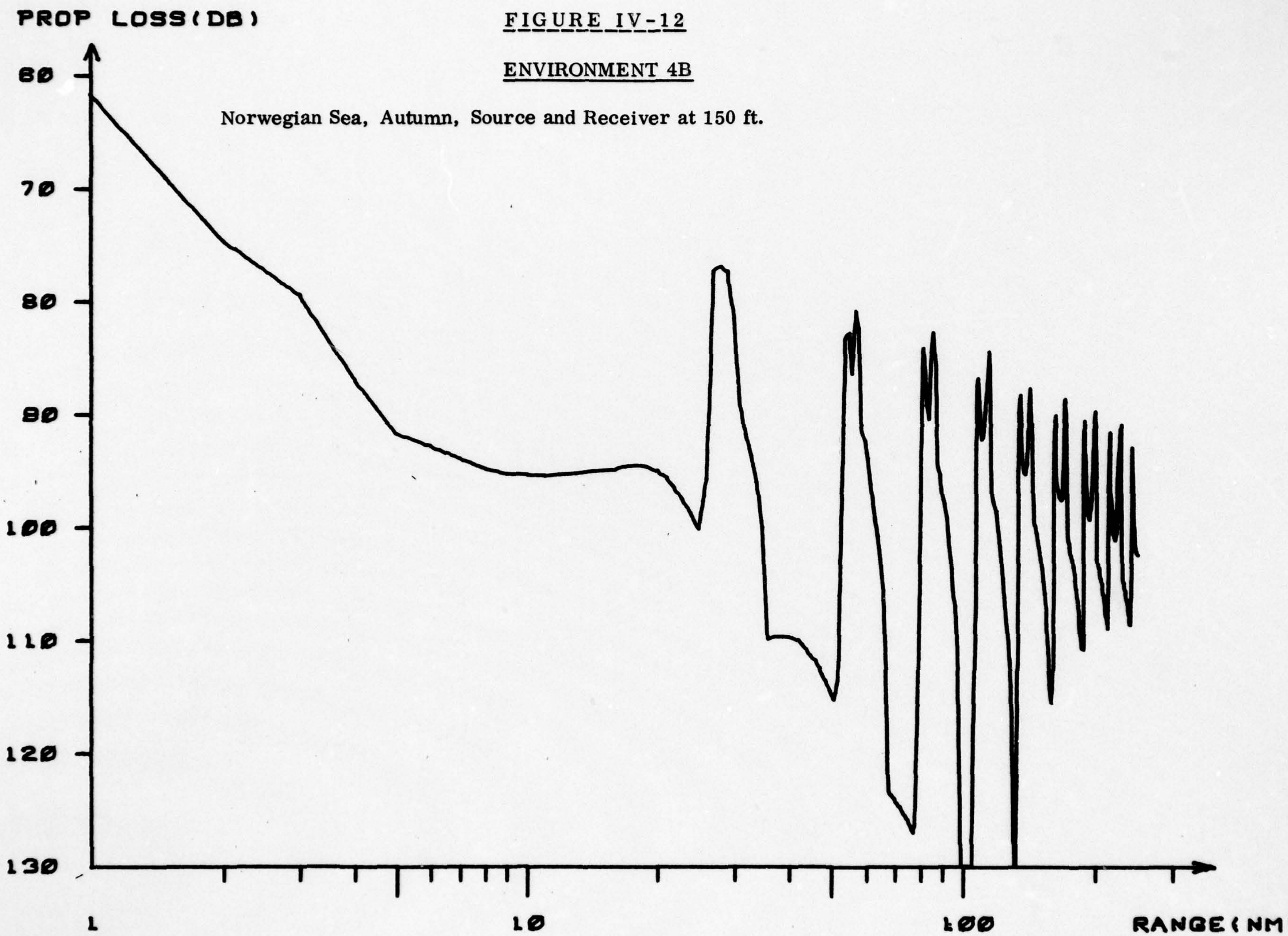
Norwegian Sea, Autumn, Source and Receiver at 150 ft.



<u>FIGURE IV-12</u>

<u>ENVIRONMENT 4B</u>

Norwegian Sea, Autumn, Source and Receiver at 150 ft.



CHAPTER V

MODELING MULTIPLE SENSORS

Finally we turn to the central problem of this report, that is, modeling the detection process for a field of sensors. We suppose there are n sensors and that all questions concerning detection are latent in the functions $x_i + \xi_i$ where x_i is the deterministic component of signal excess and ξ_i is its stochastic component with

$$\xi_i(t) = \xi_c(t) + \eta_i(t) + \eta_i'(t), \quad \text{for } t \geq 0,\ i = 1, \ldots, n.$$

We assume that ξ_c is independent of each η_i and each η_i', η_i is independent of η_k' for all i and k, and η_i' is independent of η_k' for all $i \neq k$. The components ξ_c, η_1', ..., η_n' may be modeled using the methods of reference [a], that is, as (λ, σ)-jump processes or Gauss-Markov processes or combinations thereof.

In previous chapters we have introduced a model for the correlated components $\eta_1, \ldots, \eta_n$. This is done by defining a spatial and temporal process η and taking $\eta_i(t) = \eta(P_i, t) - E\,\eta(P_i, t)$ where P_i is the location of the i^{th} sensor. We showed, moreover, that each η_i is approximately a Gauss-Markov process with autocorrelation function Ψ given by (III-1) or (III-2). This is extremely fortuitous because the η process is very cumbersome to simulate, while the Gauss-Markov process can be simulated using a simple recursive procedure which will be described below.

Our object now is to give a procedure for simulating the processes $\eta_1, \ldots, \eta_n$ such that each process η_i is a Gauss-Markov process with the required autocorrelation function Ψ and such that $\eta_i(t)$ and $\eta_k(t)$ have correlation

$$\rho_{ik} = \mathscr{C}(r_{ik})$$

where r_{ik} is the distance between sensor i and k and $\mathscr{C}$ is the spatial correlation function discussed in Chapter IV. (It should be noted that we do not have to specify the correlation between $\eta_i(t)$ and $\eta_k(s)$ for $t \neq s$, since for a Gauss-Markov process this correlation is already determined by specifying ρ_{ik} and Ψ, and is equal to $\rho_{ik}\,\Psi(t-s)$. This is a trivial consequence of the recursive relationship (V-1) below.)

Before proceeding it is necessary to mention some additional concepts. An $n \times n$ matrix of real numbers $A = ((a_{ij}))$ is said to be <u>nonnegative definite</u> if

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}\, y_j\, y_i \geq 0$$

for all real numbers $y_1, \ldots, y_n$.

If $X = (X_1, \ldots, X_n)$ is an n-dimensional random vector each of whose components has finite variance then the $n \times n$ matrix whose (i, j) component is given by

$$E(X_i - EX_i)(X_j - EX_j)$$

is called the covariance matrix of X. The matrix with (i, j) component

$$E(X_i - EX_i)(X_j - EX_j)/\sigma_i \sigma_j$$

is called the correlation matrix of X, where σ_i is the standard deviation of X_i. A correlation matrix is also a covariance matrix.

A covariance matrix $A = ((a_{ij}))$ of a random vector X is nonnegative definite because for any $y_1, \ldots, y_n$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}\, y_j\, y_i$$

$$= E\left(\sum_{i=1}^{n} y_i (X_i - EX_i)\right)^2 \geq 0.$$

If A is a nonnegative definite matrix then there exists a lower triangular square matrix B such that

$$BB^T = A.$$

(Lower triangular means all entries above the main diagonal are equal to 0; B^T means the transpose of B.) A method for computing such a B will be given below.

Returning to the main discussion, let Ω denote the $n \times n$ matrix

$$\Omega = ((\rho_{ik})) = ((\mathscr{C}(r_{ik}))).$$

This matrix is nonnegative definite because it is the correlation matrix for the random variables $\eta(P_1, t), \ldots, \eta(P_n, t)$, for any fixed t. Thus there exists an $n \times n$ matrix B such that

$$BB^T = \Omega.$$

Now let $\gamma_1, \ldots, \gamma_n$, be n independent copies of a Gauss-Markov process with covariance function $t \to \sigma^2 \exp(-\delta |t|)$, where σ^2 is the appropriate variance and $1/\delta$ is the relaxation time discussed in Chapter III. Let $\bar{\gamma}$ denote the n-dimensional process $(\gamma_1, \ldots, \gamma_n)$ and let $\bar{\eta}$ be defined by

$$\bar{\eta}^T = B\gamma^T.$$

Then the desired processes $\eta_1, \ldots, \eta_n$ may be taken as the n components of $\bar{\eta}$. That is because

$$E\bar{\eta}(t)^T \bar{\eta}(s) = E(B\gamma(t)^T)(\gamma(s)B^T)$$

$$= BE\gamma(t)^T \gamma(s) B^T$$

$$= B\sigma^2 e^{-\delta |t-s|} I B^T$$

$$= \sigma^2 e^{-\delta |t-s|} \Omega$$

where I is the $n \times n$ identity matrix. In particular each η_i has covariance function $t \to \sigma^2 e^{-\delta |t|}$ and $\eta_i(t)$ and $\eta_k(t)$ have correlation ρ_{ik} as desired.

In practice one generally needs to be able to simulate a discrete version of the Gauss-Markov process: $w(0)$, $w(\Delta)$, $w(2\Delta)$, ..., for some suitably small value of $\Delta > 0$. This is done as follows. If w should have covariance function $t \to \sigma^2 e^{-\delta |t|}$ then let $\epsilon(0)$, $\epsilon(\Delta)$, ..., be a sequence of independent draws from a Gaussian distribution with mean 0 and variance σ^2. Let w be defined recursively by:

$$w(0) = \epsilon(0),$$

and for $k \geq 1$,

$$w(k\Delta) = e^{-\delta\Delta} w\big((k-1)\Delta\big) + \sqrt{1-e^{-2\delta\Delta}}\, \epsilon\,(k\Delta). \tag{V-1}$$

Then it is easy to check that this w has the desired properties.

For the multivariate process $\bar{\eta} = (\eta_1, \ldots, \eta_n)$, one needs $\epsilon\,(k\Delta)$ to be n-dimensional multivariate Gaussian with covariance matrix $\sigma^2 I$ and one takes

$$\bar{\eta}\,(k\Delta)^T = e^{-\delta\Delta}\, \bar{\eta}\big((k-1)\Delta\big)^T + \sqrt{1-e^{-2\delta\Delta}}\; B\epsilon\,(k\Delta)^T. \tag{V-2}$$

Thus we are now in a position to simulate all the components in our model for stochastic fluctuations $\xi_i = \xi_c + \eta_i + \eta_i'$, $i = 1, \ldots, n$. The η_i' and ξ_c are all independent of each other and of the η_i, and the η_i are simulated as per the instructions above. No explicit recommendation is made here on simulating ξ_c and the η_i'; see reference [a] for some possibilities.

For moving sensors the situation is only slightly more complicated. The average velocity $\bar{v}$ in (III-1) and (III-2) is replaced by average relative velocity so that δ in (V-2) becomes time varying. The distance r_{ik} between sensor i and k becomes a time varying function so that the matrix B also becomes time varying.

We now show how, given a nonnegative definite Ω, one can compute the lower triangular matrix B such that $BB^T = \Omega$. The technique is called the Cholesky algorithm (see reference [i], p. 114).

Suppose $\Omega = \big((\rho_{ik})\big)$. Then set $b_{ik} = 0$ for all $k > i$ and compute the other b_{ik}'s by the following algorithm. (The arrow notation means: assign the value at the tail of the arrow to the variable at the head of the arrow.)

FOR k = 1 TO n

$$b_{kk} \leftarrow \left(\rho_{kk} - \sum_{j=1}^{k-1} b_{kj}^2\right)^{\frac{1}{2}}$$

FOR i = k + 1 TO n

$$b_{ik} \leftarrow \left(\rho_{ik} - \sum_{j=1}^{k-1} b_{ij}\, b_{kj}\right) / b_{kk}$$

NEXT i

NEXT k.

The entire procedure can be summarized as follows:

(i) Given the propagation loss function N_W use a Monte Carlo program to estimate the autocorrelation function $\mathscr{C}$ over a range of values which includes the minimum and maximum separation of the sensors. Use this same Monte Carlo to estimate the mean and variance of the η process.

(ii) Compute the matrix B such that $BB^T = ((\mathscr{C}(r_{ik})))$ where r_{ik} is the distance between sensor i and sensor k. B is used as described above to generate the processes $\eta_1, \ldots, \eta_n$.

(iii) For a given target path, compute the functions $x_i(k\Delta) + \xi_i(k\Delta)$ for $i = 1, \ldots, n$, and for $k = 0, 1, \ldots,$ until some inequality $x_i(k\Delta) + \xi_i(k\Delta) > 0$ is satisfied. Store this value $k\Delta$ and repeat the procedure building an empirical distribution function which estimates cdp.

Note that the computation of B is a one-time operation, <u>if</u> the sensors are stationary. For this case the simulation of the detection process is only trivially more complex than it is if we assumed that all sensors were mutually independent: the matrix operation $B\epsilon$ must be performed at each time step $k\Delta$. For moving sensors the computations in step (ii) above must be performed at every time step and this may be prohibitive except for large scale computer simulations.

LIST OF SYMBOLS USED

Symbol	Definition	Page First Used
x	deterministic component of signal excess	1
ξ	random component of signal excess	1
FOM	sonar figure of merit	1
N_W	propagation loss function	1
cdp	cumulative detection probability	1
x_i	deterministic component of signal excess at sensor i	2
ξ_i	random component of signal excess at sensor i	2
ξ_c	residual component of $\xi_1, \ldots, \xi_n$	3
η_i	correlated component of ξ_i	3
η_i'	independent component of ξ_i	3
ρ_{ik}	correlation between $\eta_i(t)$ and $\eta_k(t)$	3
$S_j(t)$	location of j^{th} noise source at time t	5
J	index set for the noise sources	5
N_j	radiated noise from the j^{th} noise source	5
θ_j	initial course of j^{th} noise source	5
v_j	speed of j^{th} noise source	5
$\eta(P, t)$	ambient noise at point P at time t	5
$d(P, Q)$	Euclidean distance between points P and Q	5
$\zeta(P, t)$	the power process, $\eta = 10 \log \zeta$	6
λ	intensity of Poisson process generating the initial positions of the noise sources	6
M	finite number of noise sources for the bounded noise process	6
$\mathscr{B}$	region in which noise sources are distributed for the bounded noise process	6

LIST OF SYMBOLS USED

Symbol	Definition	Page First Used
F_V	probability distribution for the $\{V_j\}$	6
F_N	probability distribution for the $\{N_j\}$	6
φ	characteristic function of $\zeta(P,t)$	7
Z_j	$Z_j = 10^{N_j/10}$	7
f	$f(r) = 10^{-N_W(r)/10}$, power loss factor	7
α	spreading constant	9
10β	attenuation coefficient	9
$r_j(t)$	distance of $S_j(t)$ from fixed point P	11
Ψ	autocorrelation function of the time series $t \rightarrow \eta(t)$	16
$\bar{v}$	mean of the speed distribution, $\int v\, dF_V(v)$	16
δ	$\exp(-\delta t)$ is approximately the autocorrelation function for the time series $t \rightarrow \eta(t)$; $1/\delta$ is called the relaxation time	16
$r_j(P)$	distance of $S_j(t)$ from point P at fixed time t	16
$\mathscr{C}(u)$	correlation in ambient noise between points separated by distance u	21
r_{ij}	distance between sensor i and sensor j	37
Ω	matrix of correlations $\left(\left(\mathscr{C}(r_{ij})\right)\right)$	39
B	square root of Ω, $BB^T = \Omega$	39

REFERENCES

[a] A Comparison of Detection Models Used in ASW Operations Analysis, Daniel H. Wagner, Associates Report to Office of Naval Research, by Bernard J. McCabe and Barry Belkin, dated October 31, 1973, Confidential.

[b] Commander Antisubmarine Warfare Force, Pacific, VASSEL XII Report, ser: 7/00170, Secret, August 1967.

[c] Search Pattern Assessment Model (SPAM), COMPATWINGSPAC Report, ser. 526/C69, Secret, 14 April 1976.

[d] Principles of Underwater Sound for Engineers, by R. J. Urick, McGraw-Hill, New York, 1967.

[e] "Knowledge of Acoustic Fluctuations: Who Needs It?," D. H. Wagner, Proceedings of the First Workshop on Operations Research Models of Fluctuations Affecting Passive Sonar Detection (U), David W. Taylor Naval Ship Research and Development Center Report 76-0063, January 1976 (unclassified paper in Confidential Proceedings), pp. 12-24.

[f] "Analytical Results for the Step Process and Gauss-Markov Process in Passive Detection Problems," B. Belkin, Proceedings of the First Workshop on Operations Research Models of Fluctuations Affecting Passive Sonar Detection (U), David W. Taylor Naval Ship Research and Development Center Report 76-0063, January 1976 (unclassified paper in Confidential Proceedings), pp. 199-216.

[g] Selected Papers on Noise and Stochastic Processes, edited by Nelson Wax, Dover, New York, 1954.

[h] "A Normal Limit Theorem for Power Sums of Independent Random Variables," N. A. Marlow, The Bell System Technical Journal, November 1967, pp. 2081-2089.

[i] Computer Solution of Linear Algebraic Systems, G. E. Forsythe and C. B. Moler, Prentice-Hall, Englewood Cliffs, New Jersey, 1967.

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